

# NDF Stability Analysis of Linear Networks from Return Ratios

Wayne Struble  
and  
Aryeh Platzker

## Abstract

- The purpose of this presentation is to show a method of determining the stability (or instability) of linear, passively terminated, multiport networks from the normalized network determinant function (NDF).
- This method is rigorous and can be calculated within commercially available steady-state circuit simulators.

# Outline

- Background on stability concepts
- Normalized Determinant Function (NDF)
- Use and calculation of the NDF
- Calculating the NDF from return ratios
- A ring oscillator example
- Single active element example
- NDF guidelines



# **BACKGROUND**

## Background

- Most GaAs MMIC designers use the well known Linvill or Rollett stability criteria (C or K) to determine the stability of linear two-port networks. This procedure is not rigorous!
- Platzker et al. have shown that a separate test is required to determine the stability of the network; i.e. test for the existence of any zeroes of the network determinant in the right half plane (RHP), before the Linvill or Rollett stability criteria can be applied.

## Background

- The network is stable if and only if no zeroes of the full network determinant exist in the RHP. If any zeroes do exist in the RHP, the network is unstable. This is RIGOROUS!
- Platzker's test applies the Principle of the Argument theorem to a normalized network determinant function (NDF) to determine the number of zeroes in the RHP of the full network determinant (and thus poles of the network).



# **WHAT IS THE NDF?**

## What is the NDF


- Platzker's normalized network determinant function NDF is simply the full network determinant, including all port terminations, divided by the resulting passive network determinant when all dependent sources (i.e. N voltage controlled or current controlled sources) contained within the network are set equal to zero.

$$\text{NDF} = \frac{\Delta}{\Delta_{0N}}$$



## What is the NDF

- Note that any linear network parameters such as  $Y$ ,  $Z$ ,  $H$  etc. can be used to calculate the above determinants.
- $\Delta_{0N}$  represents the determinant of a passive network and cannot contain any zeroes in the RHP. Therefore, zeroes in the RHP of the NDF correspond to zeroes in the RHP of the full network determinant.



**HOW IS THE NDF  
USED TO DETERMINE  
STABILITY?**

## Using the NDF

- To determine stability, the complex quantity NDF is calculated for a given network along the frequency axis  $\omega$  from  $-\infty$  to  $+\infty$  and its locus is plotted in the complex plane.
- If the locus of the NDF encircles the origin (0,0) in a clockwise direction, the network determinant contains zeroes in the RHP. The number of encirclements is equal to the number of zeroes in the RHP.

## Using the NDF

- A counterclockwise encirclement of the origin ( $\omega$  from  $-\infty$  to  $+\infty$ ) is not possible by construction of the NDF.
- This NDF test can be used on any physically realizable, passively terminated, linear multi-port network.

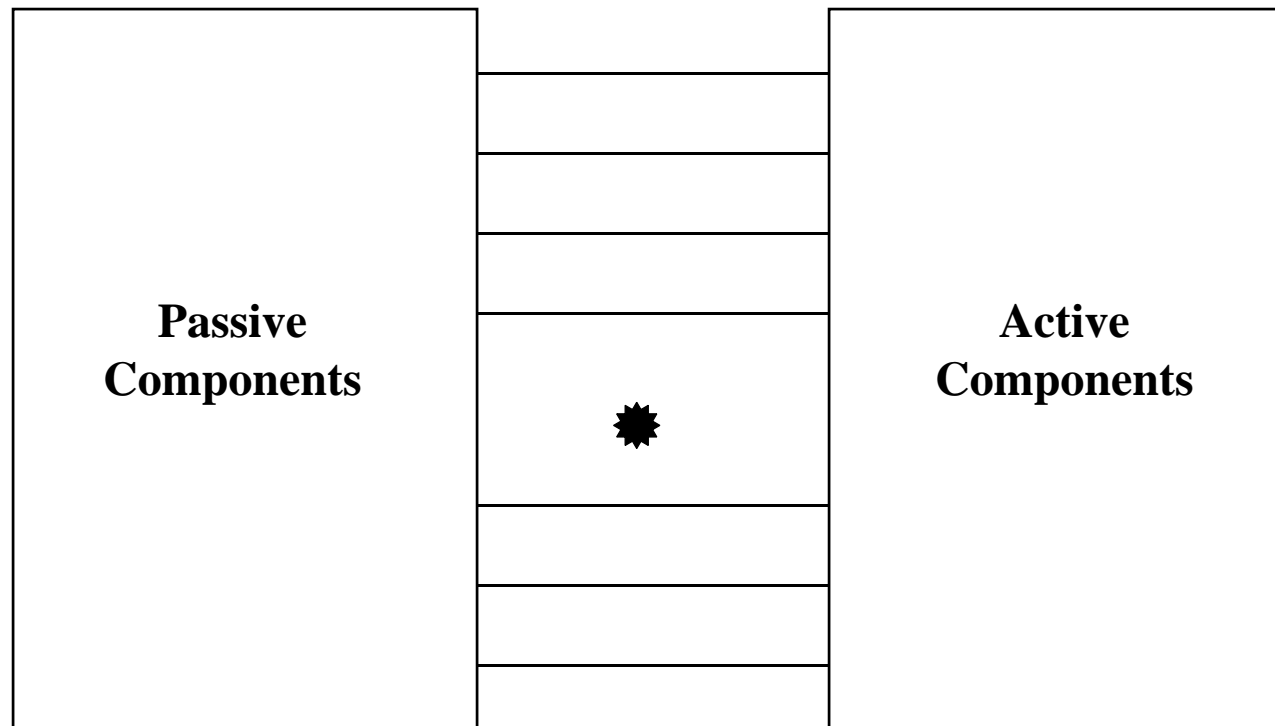


# **HOW IS THE NDF CALCULATED?**

## Calculating the NDF

- Using the brute force approach, the NDF can be found by calculating the full network determinants  $\Delta$  and  $\Delta_{0N}$  at each frequency  $\omega$ .
- A faster approach to calculating the NDF is to first reduce the network to a parallel connection of two networks, one totally passive and one totally active.

# Calculating the NDF



## Calculating the NDF

- This reduces the number of nodes in the network, and thus the sizes of the relevant matrices, to between two and four times the number of dependent sources contained in the network ( $2N-4N$ ) depending on connectivity.
- The network cannot be reduced any further than this due to the possibility of introducing poles in  $\Delta$ . This will result in pole-zero cancellation in the NDF.



## Calculating the NDF

- In networks containing more than five dependent sources, direct calculation of the NDF becomes difficult using commercially available circuit simulators.
- This is due to the limitation in the sizes of matrices that can be saved by the simulators for external calculations.
- Also, calculation of the NDF inside a circuit simulator is desirable since it allows for sensitivity analysis or optimization.

## Calculating the NDF

- To this end, we have derived an alternative way of easily calculating the NDF within commercial circuit simulators using the concept of Return Ratios.



# **A METHOD OF CALCULATING THE NDF USING RETURN RATIOS**

## NDF from Return Ratios

- The Return Ratio (RR) of a dependent source was first presented by Bode and is defined as the Return Difference minus one or,

$$\mathbf{RR} \equiv \frac{\Delta}{\Delta_0} - \mathbf{1}$$

- Where  $\Delta$  is the full network determinant and  $\Delta_0$  represents the full network determinant where the dependent source is set equal to zero.

## NDF from Return Ratios

- For networks containing a single dependent source, the Return Ratio is equivalent to the NDF and can be used as a rigorous check for stability.

$$\mathbf{NDF} = \frac{\Delta}{\Delta_{01}} = \mathbf{RR}_1 + 1$$

- If the network contains more than one dependent source,  $\Delta_{01}$  may contain zeroes in the RHP due to other dependent sources, and a single Return Ratio calculation is not sufficient for a rigorous assessment of stability.

## NDF from Return Ratios

- However, the concept can be extended to networks with N dependent sources by rearranging the previous equation into the form,

$$\Delta = (\mathbf{RR}_1 + \mathbf{1}) \cdot \Delta_{01}$$

- and realizing that,

$$\Delta_{01} = (\mathbf{RR}_2 + \mathbf{1}) \cdot \Delta_{02}$$

- Where  $\mathbf{RR}_2$  is the Return Ratio of a second dependent source in the network with the first dependent source set to zero.

## NDF from Return Ratios

- By substituting  $\Delta_{01}$  into the previous equation,

$$\Delta_{01} = (\mathbf{RR}_2 + 1) \cdot \Delta_{02} \quad \Delta = (\mathbf{RR}_1 + 1)(\mathbf{RR}_2 + 1) \cdot \Delta_{02}$$

- By continuous substitution,

$$\Delta = (\mathbf{RR}_1 + 1)(\mathbf{RR}_2 + 1)(\mathbf{RR}_3 + 1) \dots (\mathbf{RR}_N + 1) \cdot \Delta_{0N}$$


- or,

$$\mathbf{NDF} = (\mathbf{RR}_1 + 1)(\mathbf{RR}_2 + 1)(\mathbf{RR}_3 + 1) \dots (\mathbf{RR}_N + 1)$$

## NDF from Return Ratios

- For each successive Return Ratio calculation  $RR_i$  ( $i = 2-N$ ), the network is physically changed by setting all previous dependent sources to zero.





**HOW IS THE  
RETURN RATIO OF A  
DEPENDENT SOURCE  
CALCULATED?**

## Return Ratio of a Dependent Source

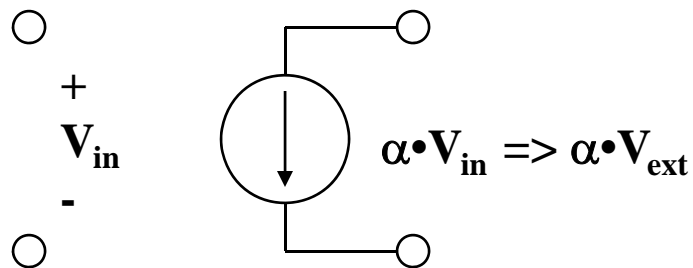
- The Return Ratio of a dependent source embedded within a network is calculated by replacing the dependent source, which is controlled by an internal voltage or current, with an identical source that is controlled by an external voltage or current.
- The stimulus from this new source will result in some amount of feedback to the controlling voltage or current of the original dependent source.

## Return Ratio of a Dependent Source

- The negative ratio of the voltage or current returned, to the external voltage or current stimulus, is the Return Ratio of the dependent source.

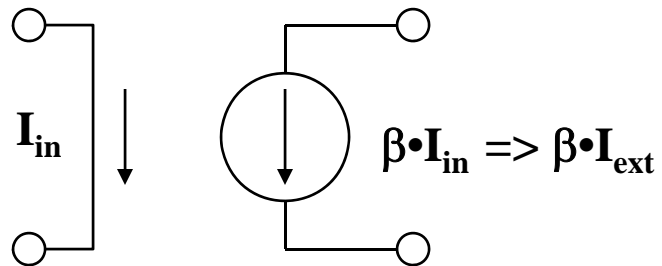
# Return Ratios of Dependent Sources

## VCCS



$$RR = \frac{-V_{\text{in}}}{V_{\text{ext}}}$$

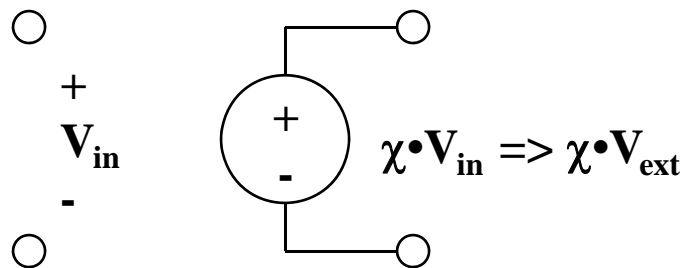
## CCCS



$$RR = \frac{-I_{\text{in}}}{I_{\text{ext}}}$$

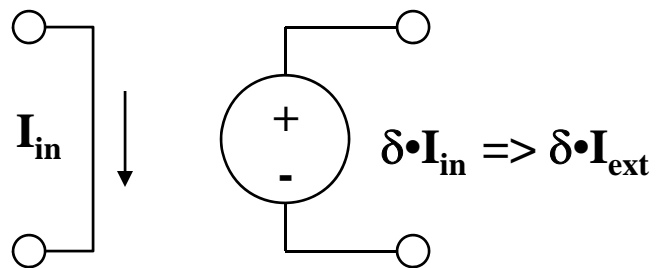
# Return Ratios of Dependent Sources

## VCVS



$$\mathbf{RR} = \frac{-V_{\text{in}}}{V_{\text{ext}}}$$

## CCVS

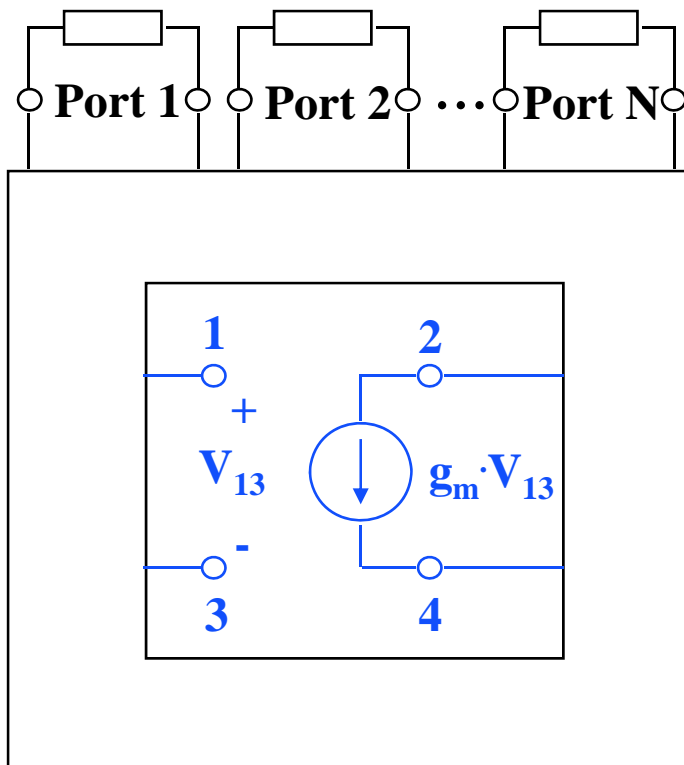


$$\mathbf{RR} = \frac{-I_{\text{in}}}{I_{\text{ext}}}$$

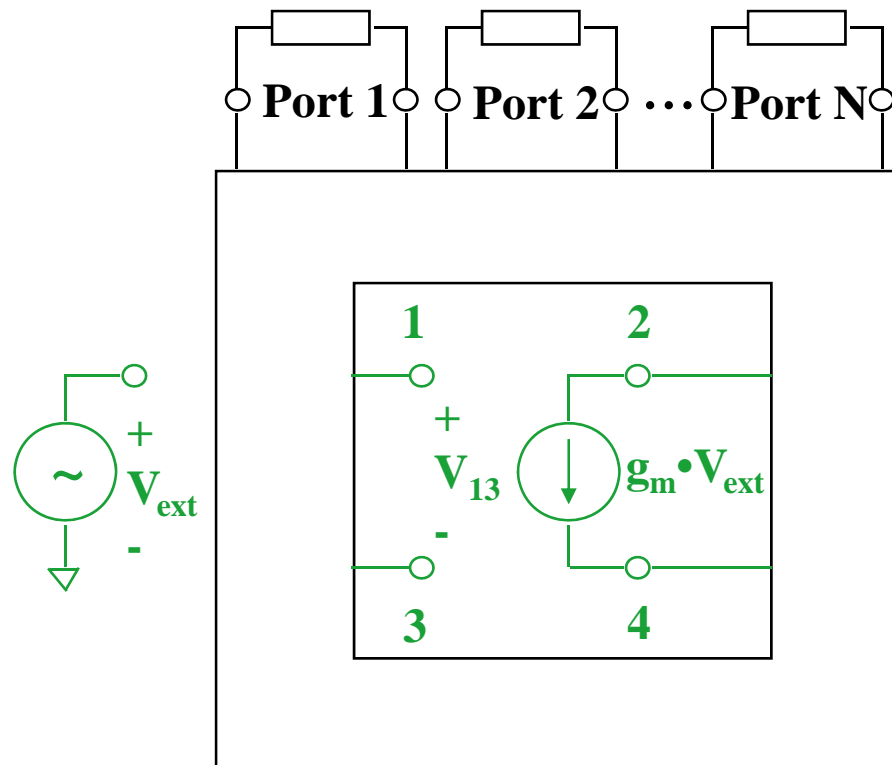
# Return Ratio of a Dependent Source

$$RR = \frac{-V_{13}}{V_{ext}}$$

**ORIGINAL NETWORK**



**RETURN RATIO NETWORK**





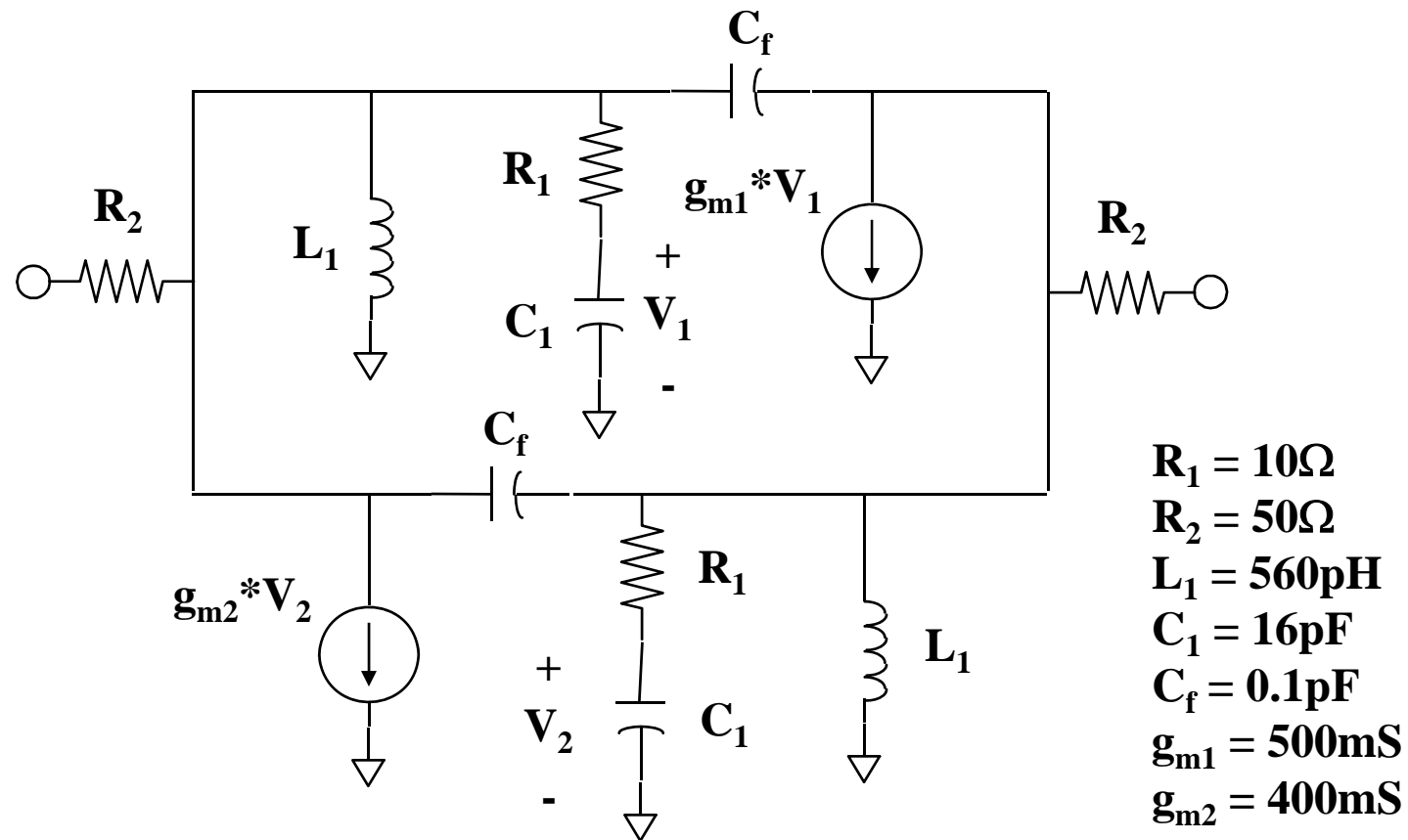
# **RING OSCILLATOR EXAMPLE**

## Ring Oscillator Example

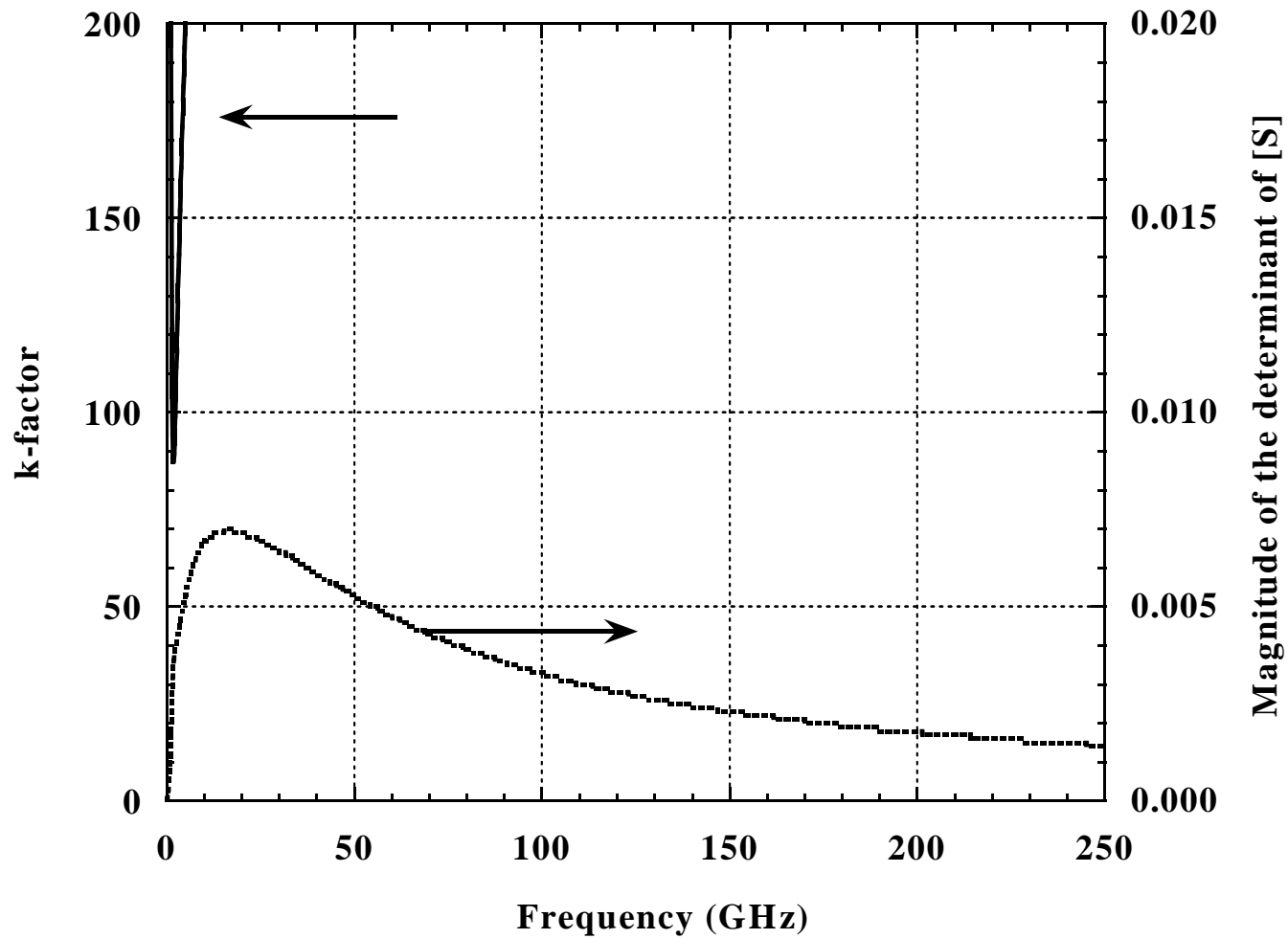
- The following example is a ring oscillator containing two dependent sources.
- The NDF of the oscillator is calculated from two Return Ratios (one Return Ratio for each dependent source in the network).
- The order of the Return Ratio calculations is unimportant as long as after calculating the Return Ratio of any given dependent source, it is set to zero in the Return Ratio calculations of all following dependent sources.



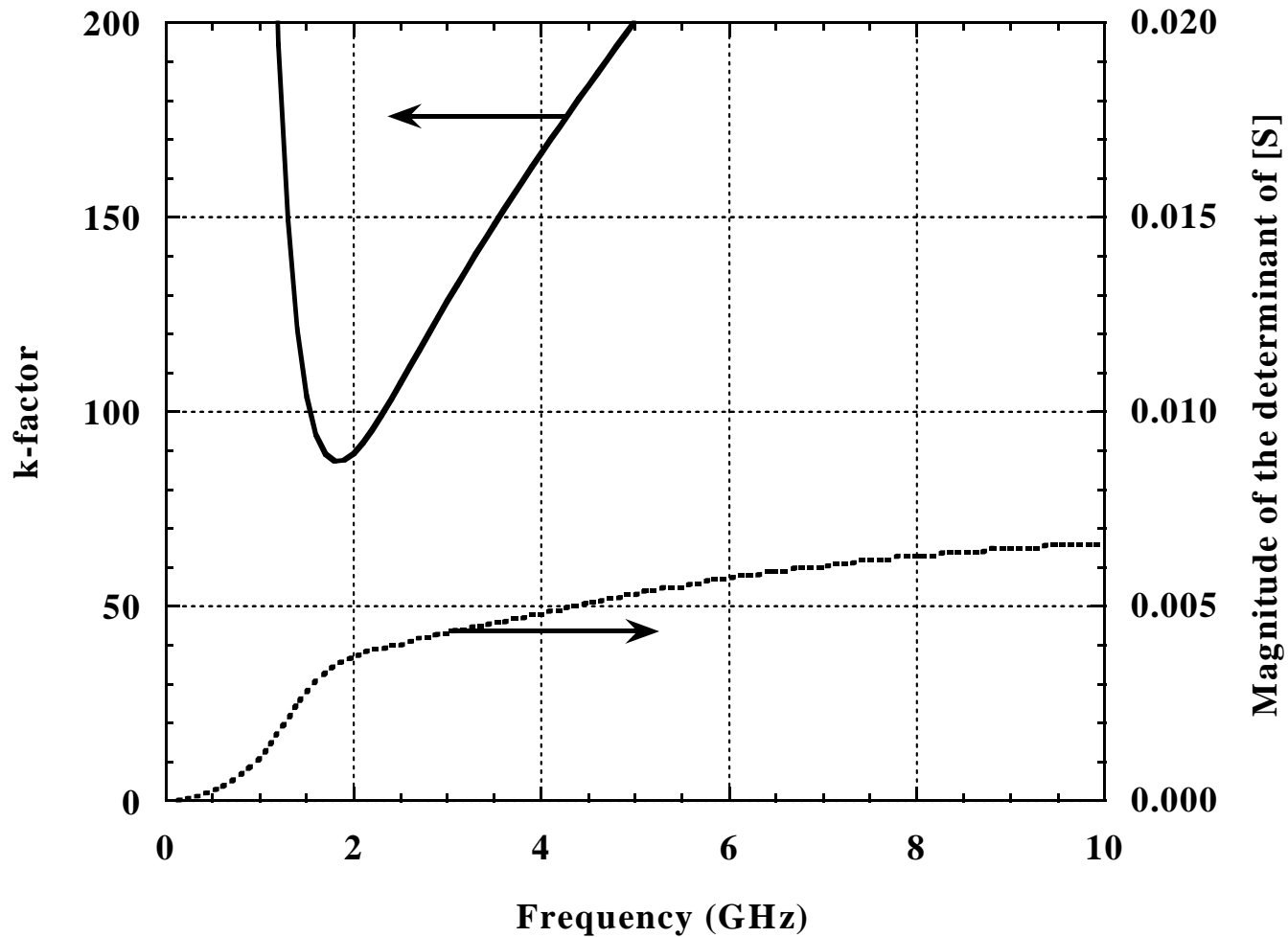
# Ring Oscillator Schematic



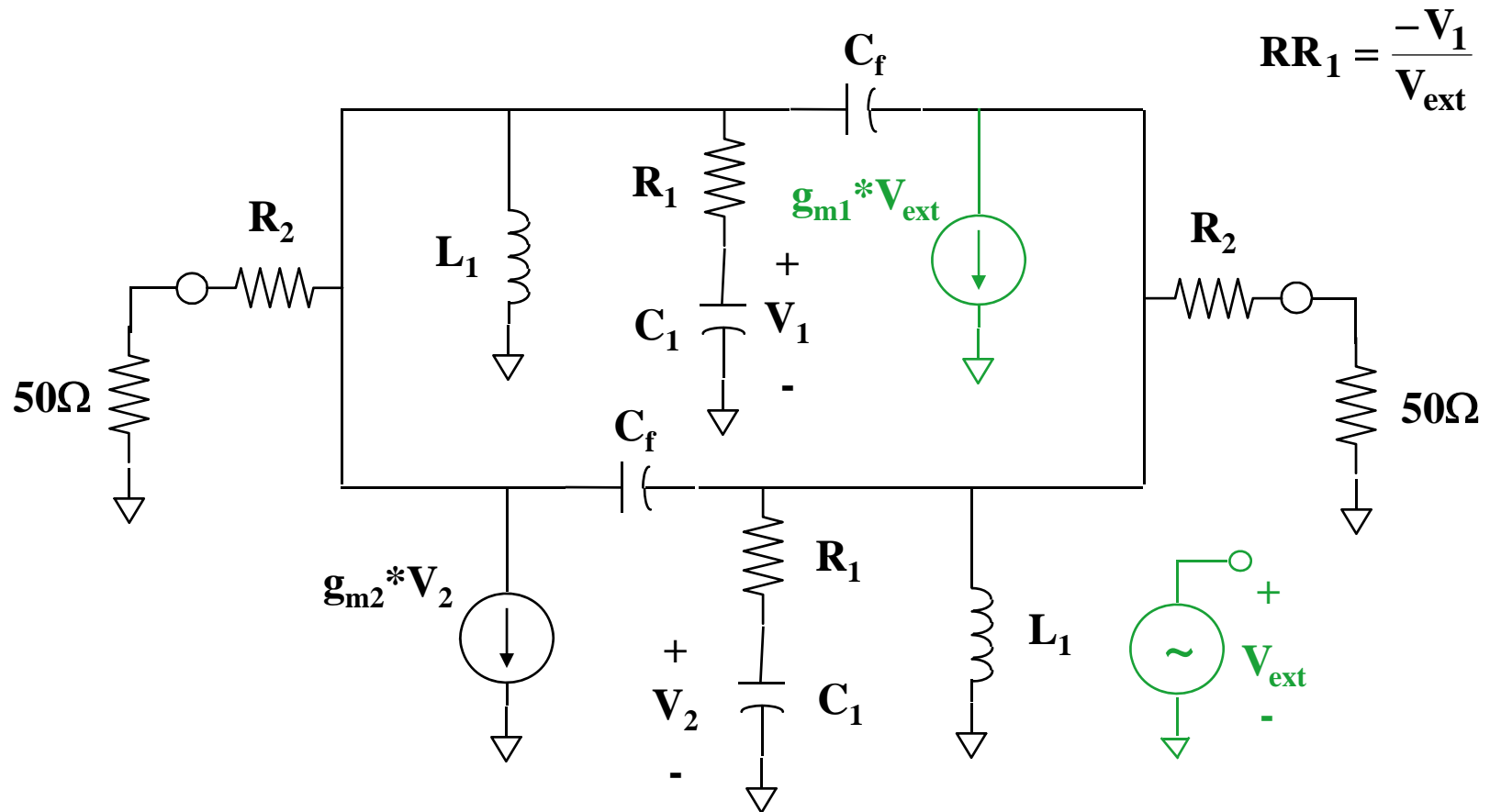
# Ring Oscillator k-factor and $|S|$



# Ring Oscillator k-factor and $|S|$

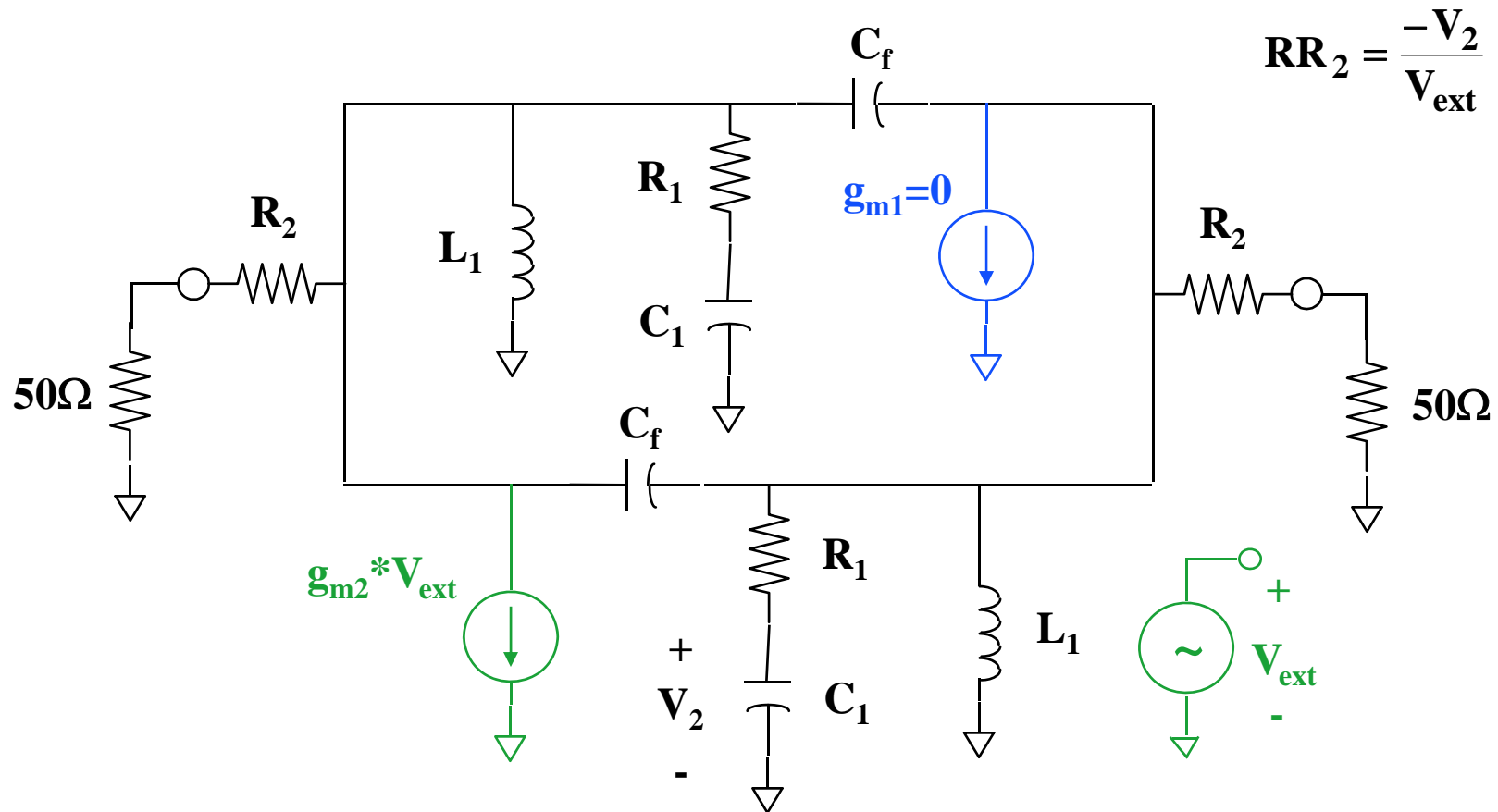


# Return Ratio $RR_1$ Schematic



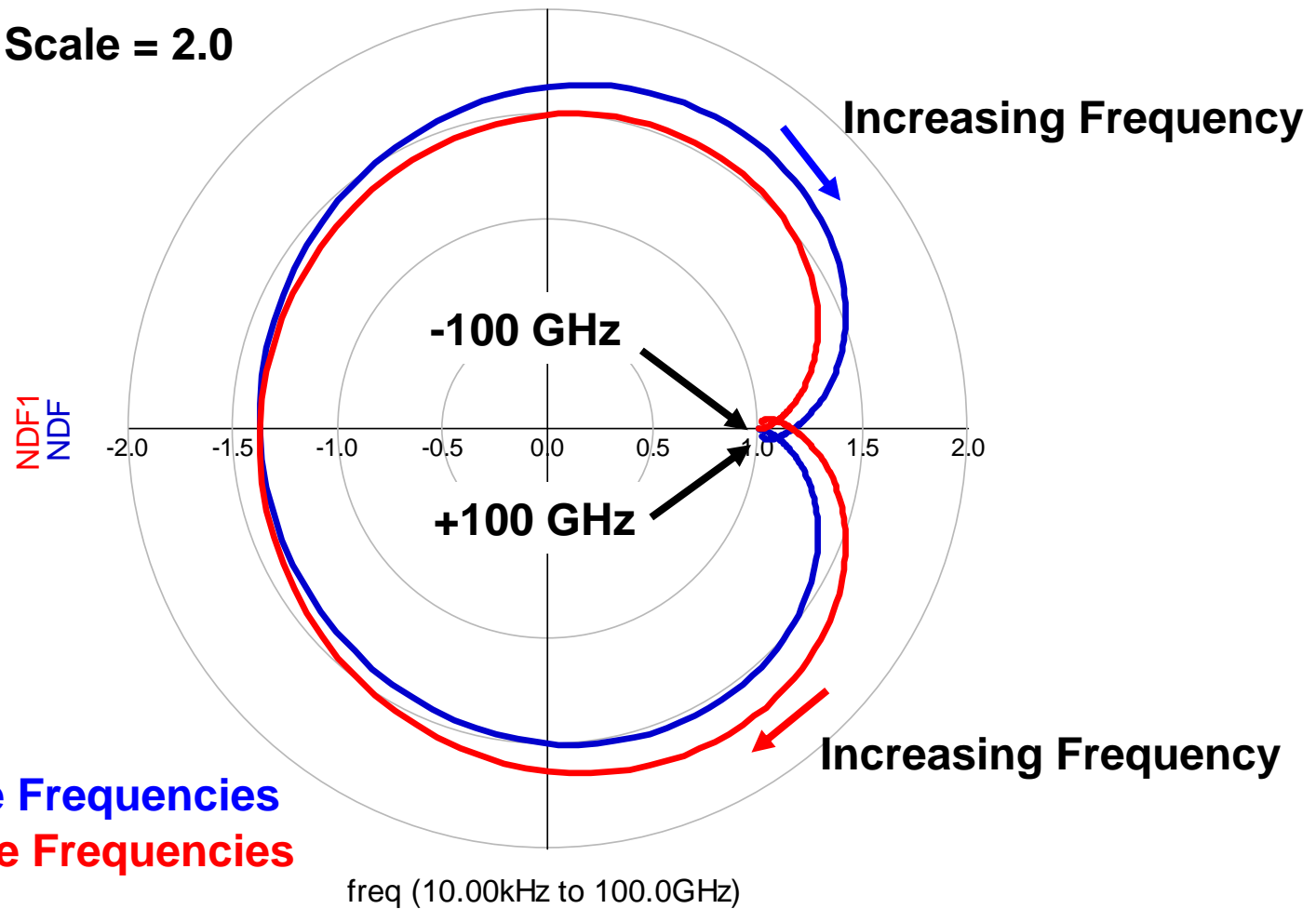
# Return Ratio $RR_2$ Schematic

$$NDF = (RR_1 + 1)(RR_2 + 1)$$

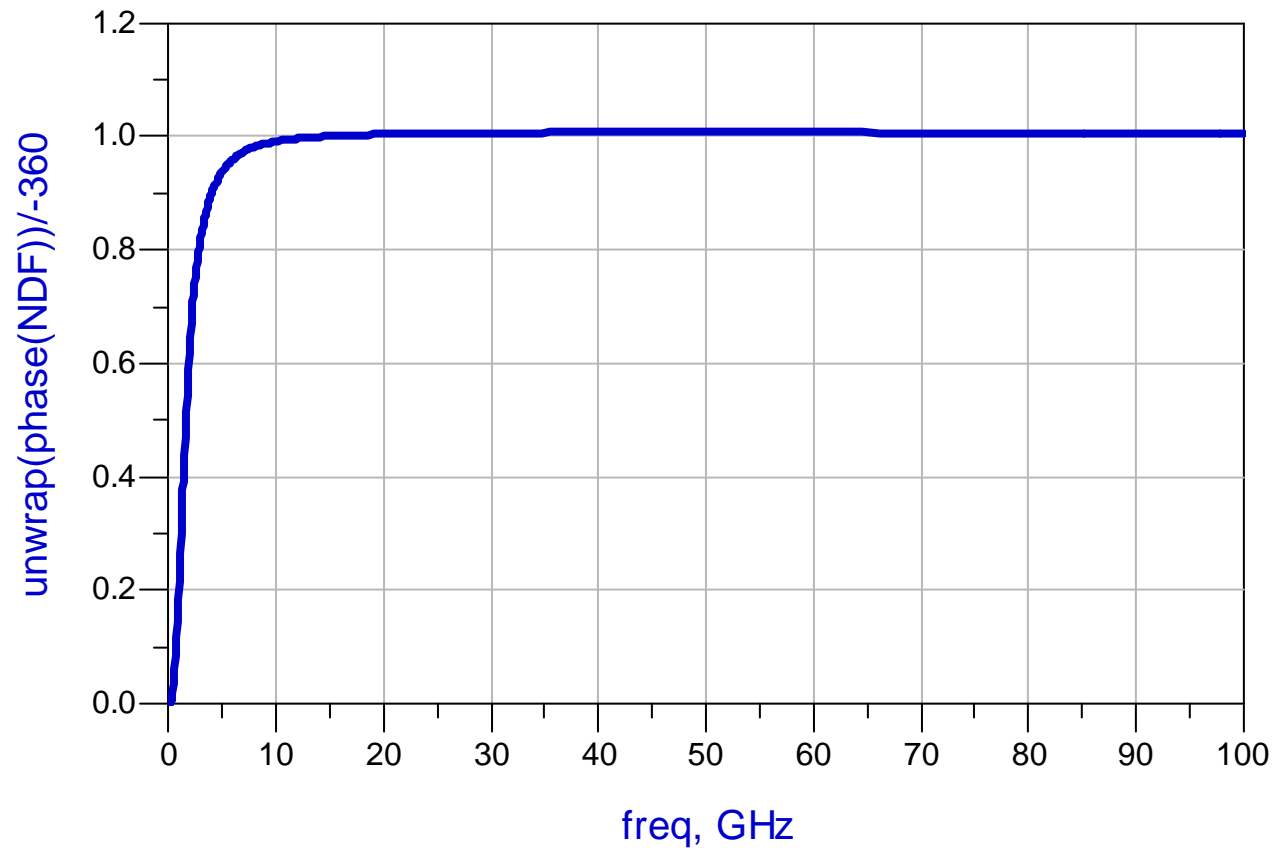


# NDF Plot of Ring Oscillator

Scale = 2.0



# NDF Encirclement Plot ( $\omega = 0$ to $+\infty$ )

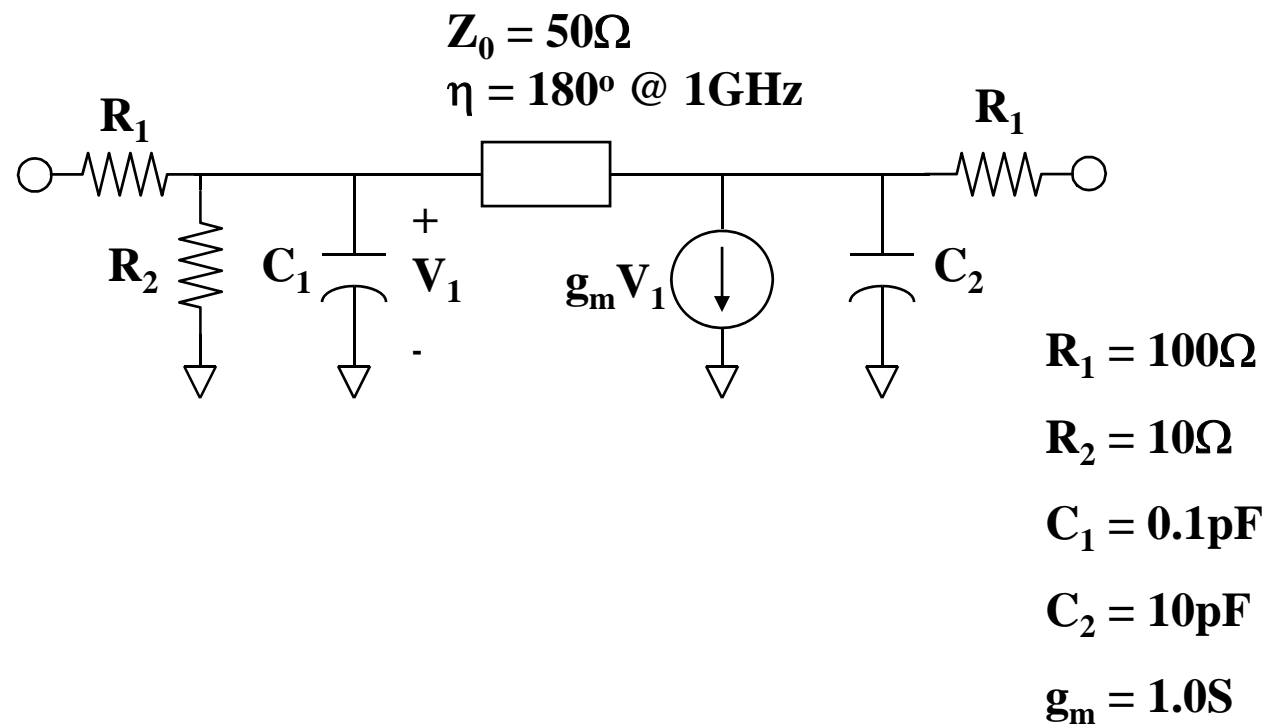




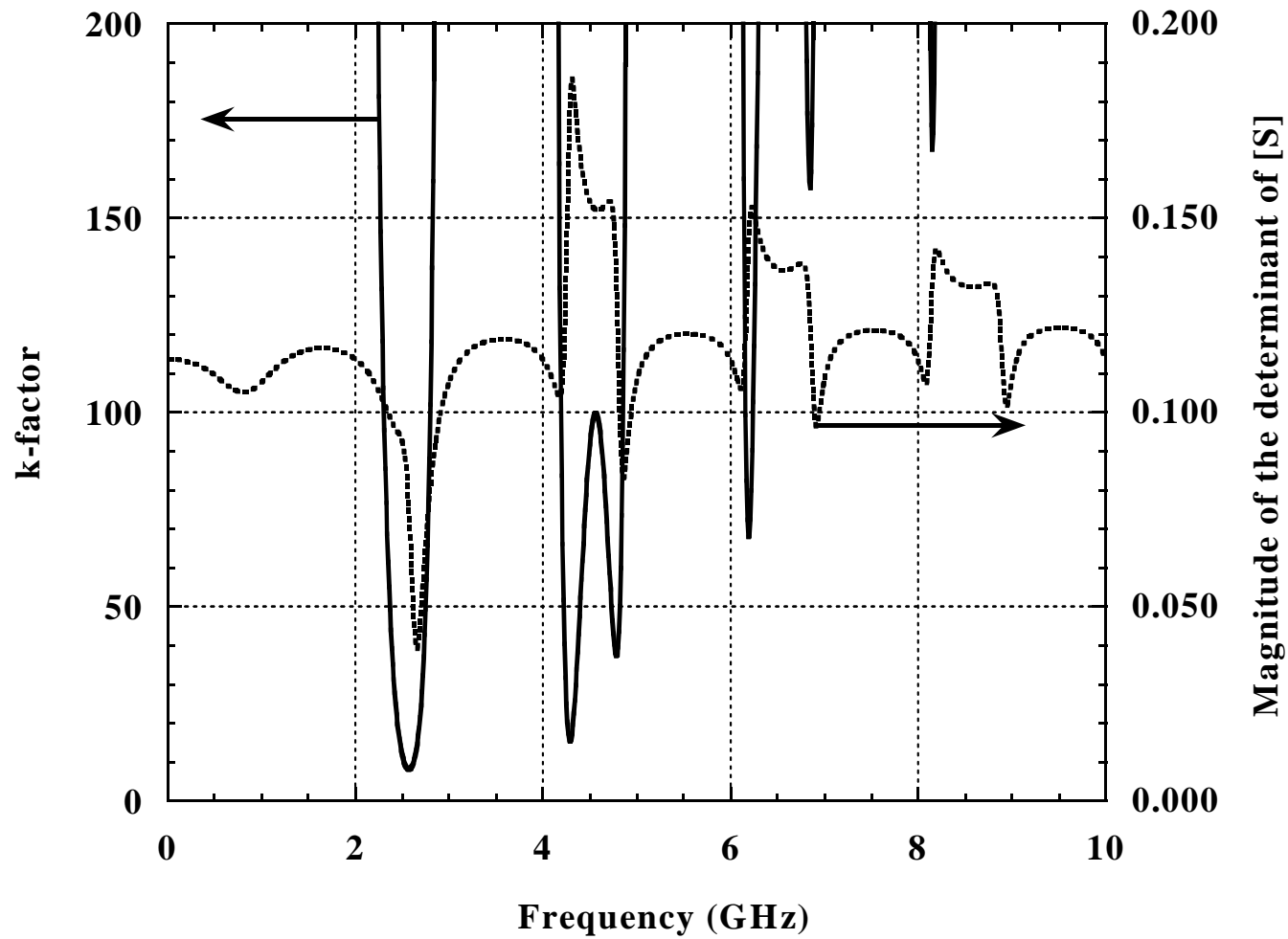
# **SINGLE ACTIVE ELEMENT EXAMPLE**



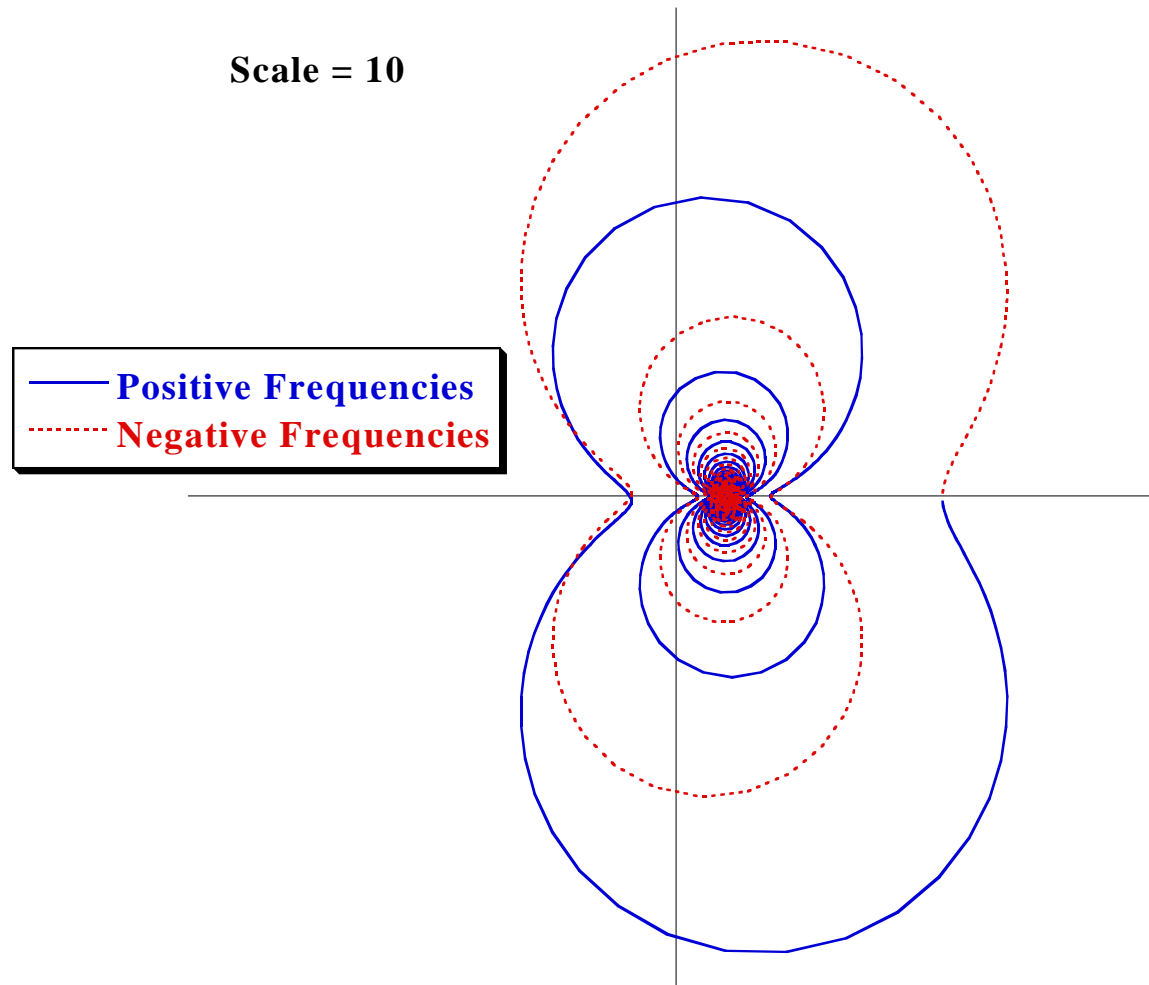
# Single Active Element Example



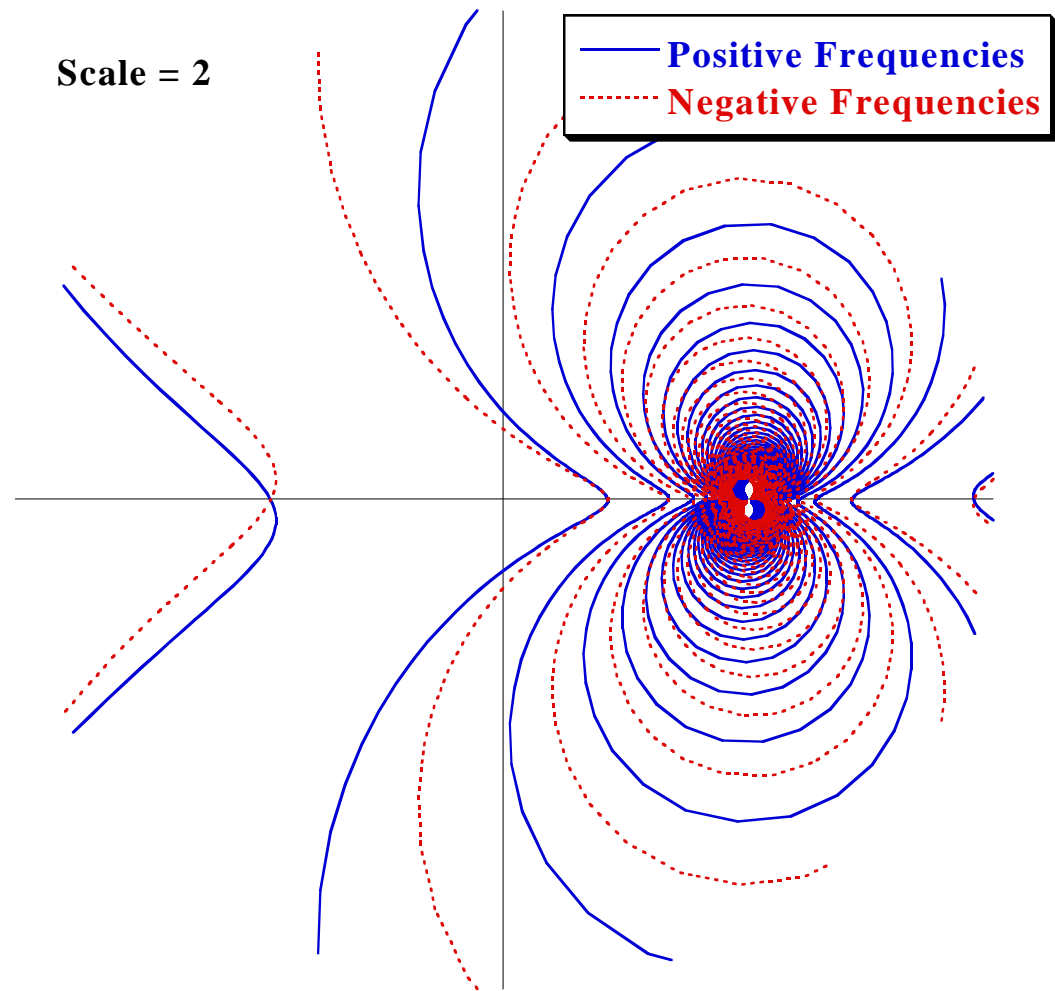
# k-factor and $|S|$



# NDF Plot (scale = 10)



# NDF Plot (scale = 2)





# **NDF GUIDELINES**

## NDF Polar Plots

- Regardless of stability, the polar plot for physically realizable networks will always begin and end at  $\omega = \pm\infty$  on the real axis at (1,0) by construction.
- The NDF does not specifically determine the frequency of oscillation (if an encirclement occurs). However, the frequency where the NDF plot crosses 180 degrees is generally close to the frequency of oscillation.

## NDF Polar Plots

- One exception to this rule is if the NDF plot touches the origin (0,0) at some frequency. If this happens, then this frequency is at least one frequency of oscillation.
- Encirclements of the origin (from  $\omega = -\infty$  to  $+\infty$ ) in a counterclockwise direction are not possible by construction of the NDF.

## NDF Encirclement Plots

- A stable network will always have a cumulative NDF phase/-360 of zero. That is, sweeping  $\omega$  from  $-\infty$  to  $+\infty$  the plot will begin (by definition) and end at zero encirclements.
- It is unimportant if the encirclement plot rises above +1 or below -1 so long as it returns to zero at  $\omega = +\infty$ .



## NDF Encirclement Plots

- A unstable network will always have a cumulative NDF phase/-360 of some multiple of 2. That is, sweeping from  $\omega = -\infty$  to  $+\infty$  the plot will begin at zero and end at +2,4,6... encirclements.
- Since the NDF over negative frequencies is the complex conjugate of positive frequencies, one can look for encirclements from  $\omega = 0$  to  $+\infty$  only. This will always show half the encirclements (as above, but capture the same stability information).
- Negative numbers of encirclements at  $\omega = +\infty$  are not possible by construction of the NDF.

# References

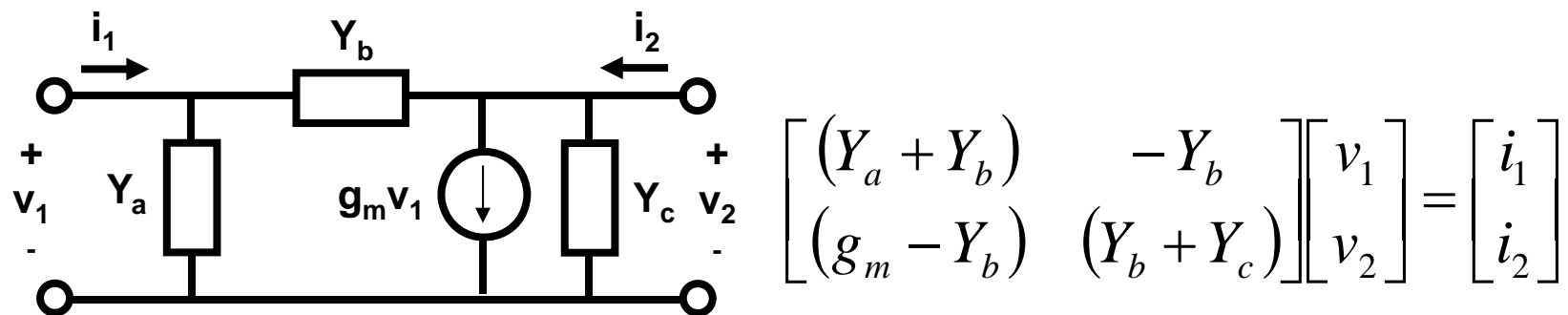
- [1] J.G. Linville and L.G. Schimpf, "The Design of Tetrode Transistor Amplifiers", Bell System Technical Journal, Vol 35, July 1956, pp. 813-840.
- [2] J.M. Rollett, "Stability and Power-Gain Invariants of Linear Two-Ports", IRE Transactions on Circuit Theory, Vol CT-9, March 1962, pp. 29-32.
- [3] H. Nyquist, "Regeneration Theory", Bell System Technical Journal, Vol 11, January 1932, pp. 126-147.
- [4] H.W. Bode, "Network Analysis and Feedback Amplifier Design", D. Van Nostrand Co. Inc., New York, 1945.
- [5] E.J. Routh, "Dynamics of a System of Rigid Bodies", 3rd Edition, Macmillan, London, 1877.
- [6] A. Platzker, W. Struble, and K. Hetzler, "Instabilities Diagnosis and the Role of K in Microwave Circuits", IEEE MTT-S International Microwave Symposium Digest, 1993, pp. 1185-1188.
- [7] W. Struble and A. Platzker, "A Rigorous Yet Simple Method for Determining Stability of Linear N-Port Networks", GaAs IC Symposium Digest, 1993, pp. 251-254.
- [8] A. Platzker and W. Struble, "Rigorous Determination of the Stability of Linear N-Node Circuits From Network Determinants and the Appropriate Role of the Stability Factor K of Their Reduced Two-Ports", 3rd International Workshop on Integrated Nonlinear Microwave and Millimeter-wave Circuits, University of Duisberg, Germany, Oct. 1994, pp. 93-107.

# Dead (passive) “black box” Transistor Models

Wayne Struble  
and  
Aryeh Platzker

## Dead “black box” Transistor Models

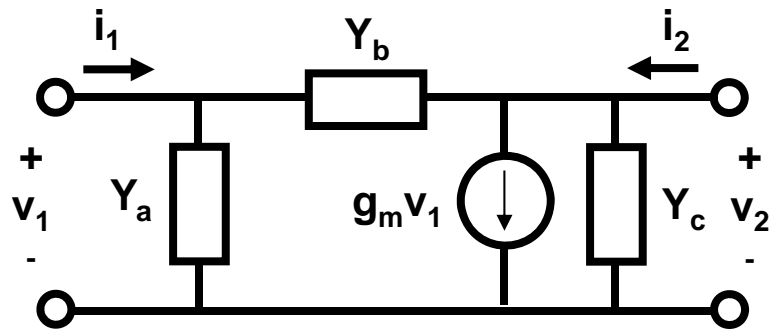
- How do we make a “black box” transistor in a linear network passive (no RHP zeroes).
- Consider the simplified network representation of an active transistor **(at any single given frequency)** (D.J.H. Maclean [1]). This is valid for any transistor type **(FET or Bipolar)**. (can also be a [S]-parameter file)



$$(Y_a + Y_b)v_1 - Y_b v_2 = i_1$$

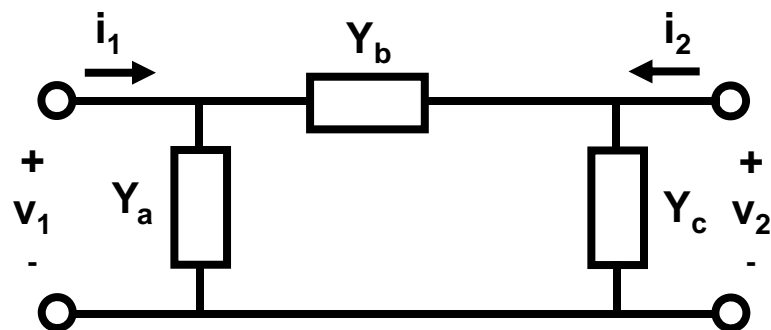
$$(g_m - Y_b)v_1 + (Y_b + Y_c)v_2 = i_2$$

## Dead “black box” Transistor Models



$$\begin{bmatrix} (Y_a + Y_b) & -Y_b \\ (g_m - Y_b) & (Y_b + Y_c) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

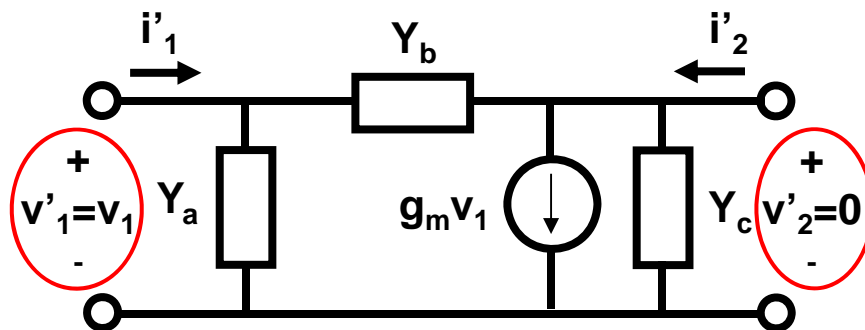
- We want to set  $g_m = 0$  at all frequencies (other than DC) to make the transistor passive.



$$\begin{bmatrix} (Y_a + Y_b) & -Y_b \\ -Y_b & (Y_b + Y_c) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

## Dead “black box” Transistor Models

- We need to subtract  $g_m v_1$  from  $i_2$  in the simplified model to achieve this. This is done using two additional (identical) transistors that use the following AC terminal conditions (DC are the same as original):
- First, in identical transistor 1, set the **input voltage** equal to  $v_1$  (sampled from the original transistor terminals), short  $v_2$  and find the short circuit current  $i_2$ .

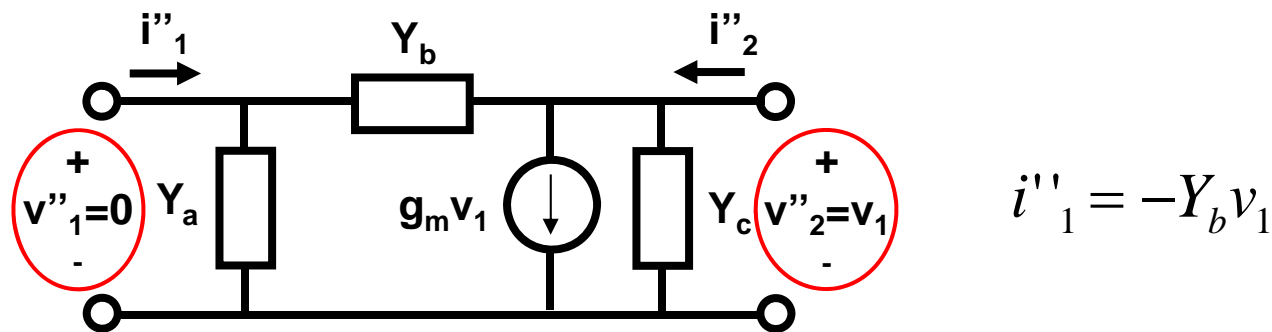


$$i'_2 = (g_m - Y_b)v_1$$

- The short circuit current is  $(g_m - Y_b)v_1$ .
- Next we need to get  $Y_b v_1$ .

## Dead “black box” Transistor Models

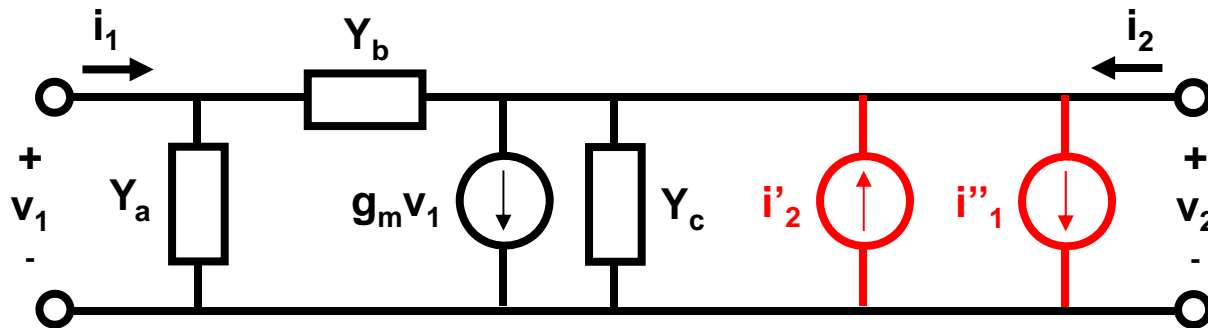
- Next, in identical transistor 2, set the **output voltage** equal to  $v_1$  (sampled from the original transistor terminals) and find the short circuit input current  $i_1$ .



- The short circuit current is  $-Y_b v_1$ .
- Next we simply subtract  $i_2'$  and add  $i_1''$  back to original transistor model as shown on the next slide.

# Dead “black box” Transistor Models

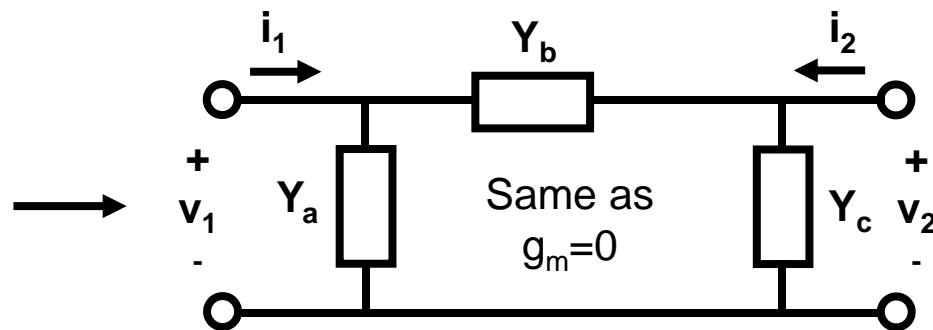
- Subtract  $i'_2$  and add  $i''_1$  to the original model like so...



$$\begin{aligned} (Y_a + Y_b)v_1 - Y_b v_2 &= i_1 & i'_2 &= (g_m - Y_b)v_1 \\ (g_m - Y_b)v_1 + (Y_b + Y_c)v_2 - i'_2 + i''_1 &= i_2 & i''_1 &= -Y_b v_1 \end{aligned}$$

This results in ↓

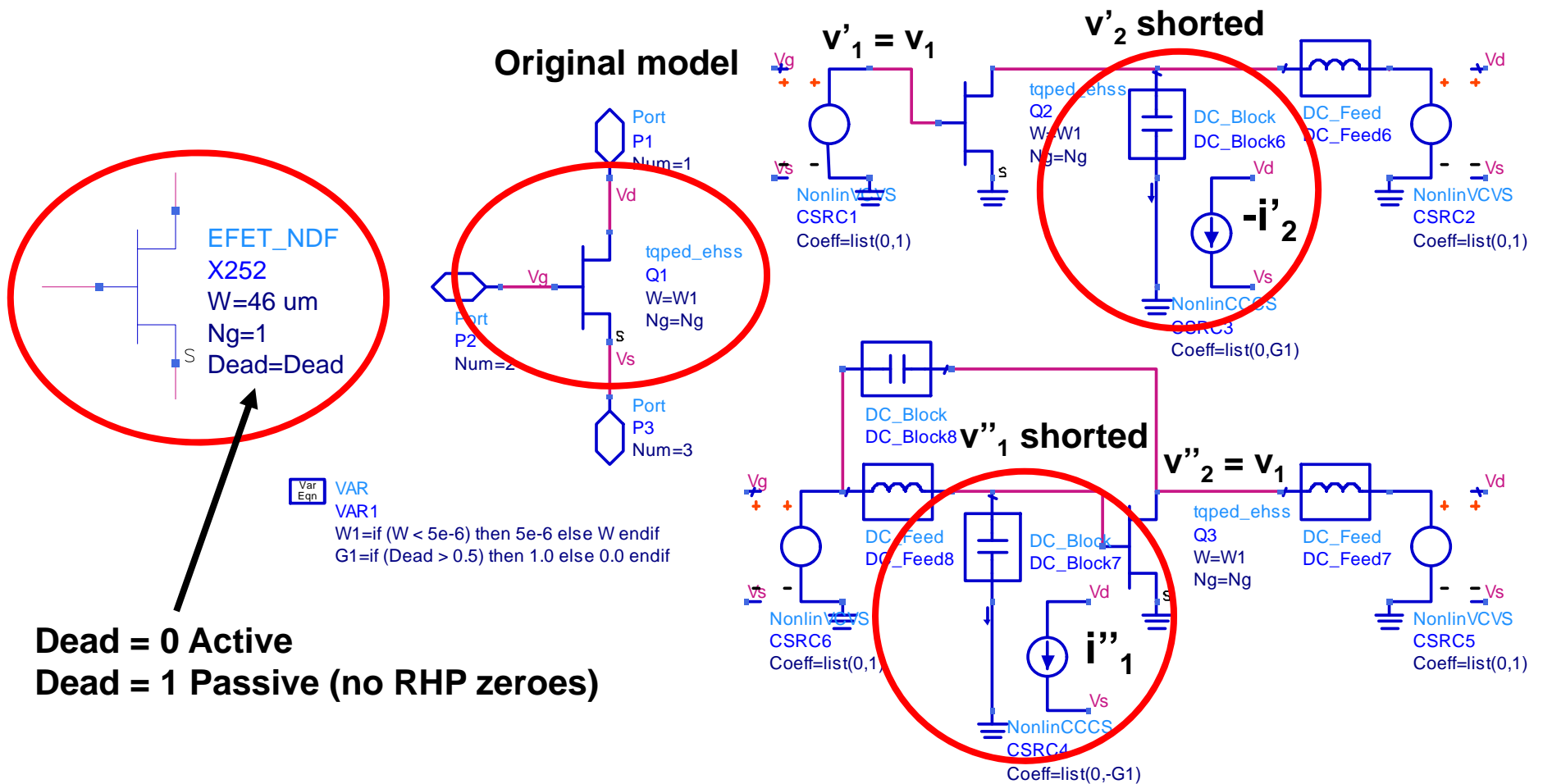
$$\begin{aligned} (Y_a + Y_b)v_1 - Y_b v_2 &= i_1 \\ -Y_b v_1 + (Y_b + Y_c)v_2 &= i_2 \end{aligned}$$





# ADS Implementation of “dead” Transistor Models (uses 3 identical transistor models)

- Model gives same DC currents as active model, but AC  $g_m = 0$  (no gain at AC frequencies). Same can be done for BJTs, [S]-parameter files.



# References

- [1] D.J.H. Maclean, "Broadband Feedback Amplifiers", Research Studies Press, 1982

# NDF Stability Analysis of Linear Networks from Current/Voltage Ratios

Wayne Struble  
and  
Aryeh Platzker

## NDF calculation using currents ratio

With a dependent current source  $g_m(v_i - v_j)$  connected between nodes k and l, and an independent source  $I_s$  connected in parallel with it, the voltage nodes can be calculated from the matrix equation using the Y matrix below

$$\begin{bmatrix}
 Y_{11} & Y_{12} & \dots & Y_{1i} & Y_{1j} & \dots & Y_{1k} & Y_{1l} & \dots & Y_{1n} \\
 Y_{21} & Y_{22} & \dots & Y_{2i} & Y_{2j} & \dots & Y_{2k} & Y_{2l} & \dots & Y_{2n} \\
 \cdot & & & \cdot & \cdot & & \cdot & \cdot & & \cdot \\
 \cdot & & & \cdot & \cdot & & \cdot & \cdot & & \cdot \\
 Y_{k1} & Y_{k2} & \dots & -g_m + Y_{ki} & Y_{kj} + g_m & \dots & Y_{kk} & Y_{kl} & \dots & Y_{kn} \\
 Y_{l1} & Y_{l2} & \dots & g_m + Y_{li} & Y_{lj} - g_m & \dots & Y_{lk} & Y_{ll} & \dots & Y_{ln} \\
 \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot \\
 Y_{n1} & Y_{n2} & \dots & Y_{ni} & Y_{nj} & \dots & Y_{nk} & Y_{nl} & & Y_{nn}
 \end{bmatrix}
 \begin{bmatrix}
 v_1 \\
 v_2 \\
 \cdot \\
 \cdot \\
 v_k \\
 v_l \\
 \cdot \\
 v_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 -I_s \\
 I_s \\
 0 \\
 0
 \end{bmatrix}$$

# NDF calculation using currents ratio

Introduce the sum of the independent and the dependent currents  $I$  as a new variable  $I = I_s - g_m (V_i - V_j)$  and obtain the circuit description with the augmented  $Y$  matrix,  $Y_{aug}$

$$\begin{bmatrix}
 Y_{11} & Y_{12} & \dots & Y_{1i} & Y_{1j} & \dots & Y_{1k} & Y_{1l} & \dots & Y_{1n} & 0 \\
 Y_{21} & Y_{22} & \dots & Y_{2i} & Y_{2j} & \dots & Y_{2k} & Y_{2l} & \dots & Y_{2n} & 0 \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 Y_{k1} & Y_{k2} & \dots & Y_{ki} & Y_{kj} & \dots & Y_{kk} & Y_{kl} & \dots & Y_{kn} & 1 \\
 Y_{l1} & Y_{l2} & \dots & Y_{li} & Y_{lj} & \dots & Y_{lk} & Y_{ll} & \dots & Y_{ln} & -1 \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 Y_{n1} & Y_{n2} & \dots & Y_{ni} & Y_{nj} & \dots & Y_{nk} & Y_{nl} & \dots & Y_{nn} & 0 \\
 0 & 0 & \dots & g_m & -g_m & \dots & 0 & 0 & \dots & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 v_1 \\
 v_2 \\
 \cdot \\
 v_k \\
 v_l \\
 \cdot \\
 v_n \\
 I
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 \cdot \\
 0 \\
 0 \\
 \cdot \\
 0 \\
 I_s
 \end{bmatrix}$$

# NDF calculation using currents ratio

By Cramer's rule the variable  $I$  is given by

$$|Y_{aug}| I = \begin{vmatrix} Y_{11} & Y_{12} & \dots & Y_{1i} & Y_{1j} & \dots & Y_{1k} & Y_{1l} & \dots & Y_{1n} & 0 \\ Y_{21} & Y_{22} & \dots & Y_{2i} & Y_{2j} & \dots & Y_{2k} & Y_{2l} & \dots & Y_{2n} & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{k1} & Y_{k2} & \dots & Y_{ki} & Y_{kj} & \dots & Y_{kk} & Y_{kl} & \dots & Y_{kn} & 0 \\ Y_{l1} & Y_{l2} & \dots & Y_{li} & Y_{lj} & \dots & Y_{lk} & Y_{ll} & \dots & Y_{ln} & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{n1} & Y_{n2} & \dots & Y_{ni} & Y_{nj} & \dots & Y_{nk} & Y_{nl} & \dots & Y_{nn} & 0 \\ 0 & 0 & \dots & g_m & -g_m & \dots & 0 & 0 & \dots & 0 & I_s \end{vmatrix}$$

From which  $I_s / I = |Y_{aug}| / |Y_{aug}(g_m=0)|$

i.e.  $I_s / I$  is the Normalized Determinant Function (NDF) of the circuit

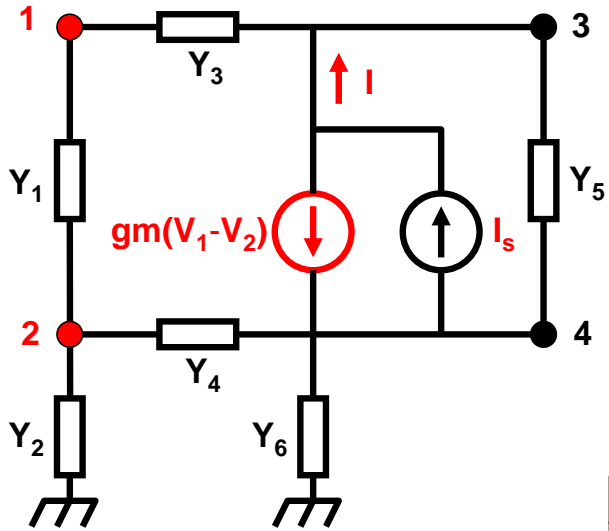
## NDF calculation using currents ratio

It is easy to show that  $|Y| = |Y_{aug}|$  by observing that subtracting in  $Y_{aug}$  its  $k$ 'th row from its  $(n+1)$ 'th row and adding its  $l$ 'th row to its  $(n+1)$ 'th row leads  $Y_{aug}$  to become:

$$\begin{array}{cccccccccc}
 Y_{11} & Y_{12} & \dots & Y_{1i} & Y_{1j} & \dots & Y_{1k} & Y_{1l} & \dots & Y_{1n} & 0 \\
 Y_{21} & Y_{22} & \dots & Y_{2i} & Y_{2j} & \dots & Y_{2k} & Y_{2l} & \dots & Y_{2n} & 0 \\
 \cdot & & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\
 \cdot & & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\
 Y_{k1} & Y_{k2} & \dots & -g_m + Y_{ki} & Y_{kj} + g_m & \dots & Y_{kk} & Y_{kl} & \dots & Y_{kn} & 0 \\
 Y_{l1} & Y_{l2} & \dots & g_m + Y_{li} & Y_{lj} - g_m & \dots & Y_{lk} & Y_{ll} & \dots & Y_{ln} & 0 \\
 \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\
 Y_{n1} & Y_{n2} & \dots & Y_{ni} & Y_{nj} & \dots & Y_{nk} & Y_{nl} & \dots & Y_{nn} & 0 \\
 0 & 0 & & g_m & -g_m & & 0 & 0 & & 0 & 1
 \end{array}$$

Since the row adding operations we performed do not change the value of the determinant, we readily see by expanding the determinant along the  $(n+1)$ 'th column that  $|Y| = |Y_{aug}|$

# NDF calculation using currents ratio



$$\begin{aligned}
 Y_1(V_1 - V_2) + Y_3(V_1 - V_3) &= 0 \\
 Y_1(V_2 - V_1) + Y_2V_2 + Y_4(V_2 - V_4) &= 0 \\
 Y_3(V_3 - V_1) + Y_5(V_3 - V_4) + I &= 0 \\
 Y_4(V_4 - V_2) + Y_5(V_4 - V_3) + Y_6V_4 - I &= 0 \\
 gm(V_1 - V_2) + I &= I_s
 \end{aligned}$$

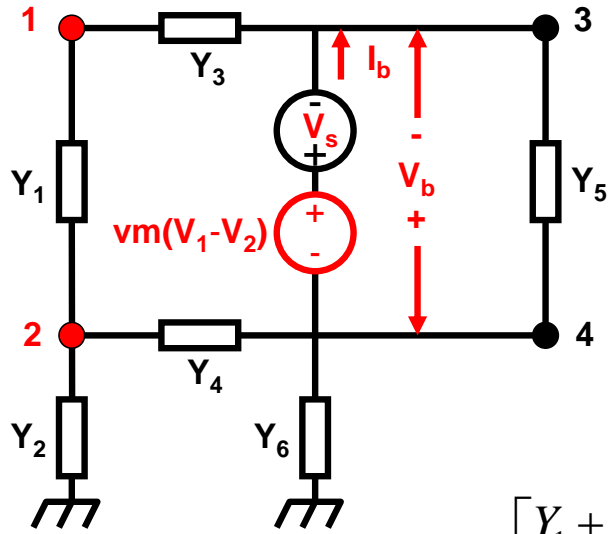
$$\begin{bmatrix}
 Y_1 + Y_3 & -Y_1 & -Y_3 & 0 & 0 \\
 -Y_1 & Y_1 + Y_2 + Y_4 & 0 & -Y_4 & 0 \\
 -Y_3 & 0 & Y_3 + Y_5 & -Y_5 & 1 \\
 0 & -Y_4 & Y_5 & Y_4 + Y_5 + Y_6 & -1 \\
 gm & -gm & 0 & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 V_1 \\
 V_2 \\
 V_3 \\
 V_4 \\
 I
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 I_s
 \end{bmatrix}$$

By Cramer's Rule we get:

$$NDF = \frac{I_s}{I} = \frac{|Y|}{|Y|_{(gm=0)}}$$



# NDF calculation using voltages ratio



$$\begin{aligned}
 Y_1(V_1 - V_2) + Y_3(V_1 - V_3) &= 0 \\
 Y_1(V_2 - V_1) + Y_2V_2 + Y_4(V_2 - V_4) &= 0 \\
 Y_3(V_3 - V_1) + Y_5(V_3 - V_4) - I_b &= 0 \\
 Y_4(V_4 - V_2) + Y_5(V_4 - V_3) + Y_6V_4 + I_b &= 0 \\
 (V_4 - V_3) - V_b &= 0 \\
 vm(V_1 - V_2) + V_b &= V_s
 \end{aligned}$$

$$\begin{bmatrix}
 Y_1 + Y_3 & -Y_1 & -Y_3 & 0 & 0 & 0 \\
 -Y_1 & Y_1 + Y_2 + Y_4 & 0 & -Y_4 & 0 & 0 \\
 -Y_3 & 0 & Y_3 + Y_5 & -Y_5 & -1 & 0 \\
 0 & -Y_4 & -Y_5 & Y_4 + Y_5 + Y_6 & 1 & 0 \\
 0 & 0 & -1 & 1 & 0 & -1 \\
 vm & -vm & 0 & 0 & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 V_1 \\
 V_2 \\
 V_3 \\
 V_4 \\
 I_b \\
 V_b
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 V_s
 \end{bmatrix}$$

By Cramer's Rule we get:

$$NDF = \frac{V_s}{V_b} = \frac{|Y|}{|Y|_{(vm=0)}}$$

# Cramer's Rule

(from Wolfram Math World)

Given a set of linear equations

$$\begin{cases} a_1 x + b_1 y + c_1 z = d_1 \\ a_2 x + b_2 y + c_2 z = d_2 \\ a_3 x + b_3 y + c_3 z = d_3, \end{cases} \quad (1)$$

consider the **determinant**

$$D \equiv \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}. \quad (2)$$

Now multiply  $D$  by  $x$ , and use the property of **determinants** that **multiplication** by a constant is equivalent to **multiplication** of each entry in a single column by that constant, so

$$x \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 x & b_1 & c_1 \\ a_2 x & b_2 & c_2 \\ a_3 x & b_3 & c_3 \end{vmatrix}. \quad (3)$$

Another property of **determinants** enables us to add a constant times any column to any column and obtain the same **determinant**, so add  $y$  times column 2 and  $z$  times column 3 to column 1,

$$x D = \begin{vmatrix} a_1 x + b_1 y + c_1 z & b_1 & c_1 \\ a_2 x + b_2 y + c_2 z & b_2 & c_2 \\ a_3 x + b_3 y + c_3 z & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}. \quad (4)$$

If  $\mathbf{d} = \mathbf{0}$ , then (4) reduces to  $x D = 0$ , so the system has nondegenerate solutions (i.e., solutions other than  $(0, 0, 0)$ ) only if  $D = 0$  (in which case there is a family of solutions). If  $\mathbf{d} \neq \mathbf{0}$  and  $D = 0$ , the system has no unique solution. If instead  $\mathbf{d} \neq \mathbf{0}$  and  $D \neq 0$ , then solutions are given by

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}, \quad (5)$$

and similarly for

$$y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D} \quad (6)$$

$$z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}. \quad (7)$$

This procedure can be generalized to a set of  $n$  equations so, given a system of  $n$  linear equations

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}, \quad (8)$$

let

$$D \equiv \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}. \quad (9)$$

If  $\mathbf{d} = \mathbf{0}$ , then nondegenerate solutions exist only if  $D = 0$ . If  $\mathbf{d} \neq \mathbf{0}$  and  $D = 0$ , the system has no unique solution. Otherwise, compute

$$D_k \equiv \begin{vmatrix} a_{11} & \cdots & a_{1(k-1)} & d_1 & a_{1(k+1)} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{n(k-1)} & d_n & a_{n(k+1)} & \cdots & a_{nn} \end{vmatrix}. \quad (10)$$

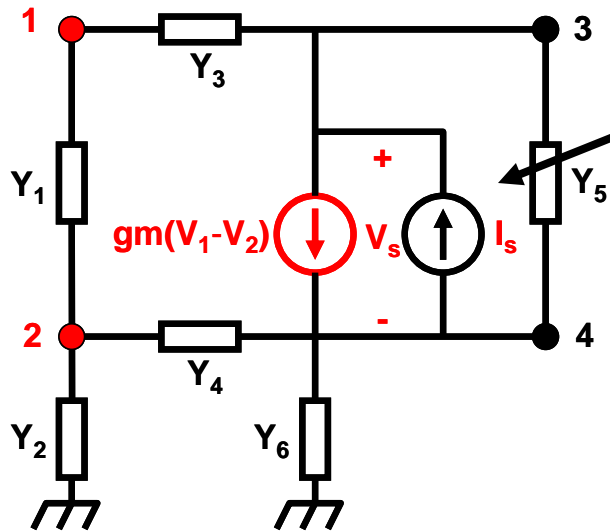
Then  $x_k = D_k / D$  for  $1 \leq k \leq n$ . In the three-dimensional case, the [vector](#) analog of Cramer's rule is

$$(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B} \times \mathbf{D}) \mathbf{C} - (\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}) \mathbf{D}. \quad (11)$$

# **NDF Stability Analysis of Linear Networks from Network Admittances**

**Wayne Struble  
and  
Aryeh Platzker**

## Relationship Between Network Determinants and Admittance



Add current source  $I_s$   
in parallel with dependent  
Source and measure  $V_s$

$$\text{Admittance} = I_s / V_s$$

$$[Y][V] = [I]$$

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & 0 & -Y_4 \\ gm - Y_3 & -gm & Y_3 + Y_5 & -Y_5 \\ -gm & gm - Y_4 & -Y_5 & Y_4 + Y_5 + Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_s \\ -I_s \end{bmatrix}$$

## Relationship Between Network Determinants and Admittance

- First redefine the node voltages in terms of  $V_s$  where  $V_4 = V_3 - V_s$
- So, column 3 in the matrix becomes the sum of columns 3 and 4, and column 4 changes sign as shown below.

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 & Y_4 \\ gm - Y_3 & -gm & Y_3 & Y_5 \\ -gm & gm - Y_4 & Y_4 + Y_6 & -Y_4 - Y_5 - Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_s \\ -I_s \end{bmatrix}$$

## Relationship Between Network Determinants and Admittance

- Next, we replace row 3 in the matrix with the sum of rows 3 and 4 to eliminate  $I_s$  on the right side.

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 & Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 & -Y_4 - Y_6 \\ -gm & gm - Y_4 & Y_4 + Y_6 & -Y_4 - Y_5 - Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_s \end{bmatrix}$$

- And multiplying row 4 by -1,

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 & Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 & -Y_4 - Y_6 \\ gm & -gm + Y_4 & -Y_4 - Y_6 & Y_4 + Y_5 + Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_s \end{bmatrix}$$



## Relationship Between Network Determinants and Admittance

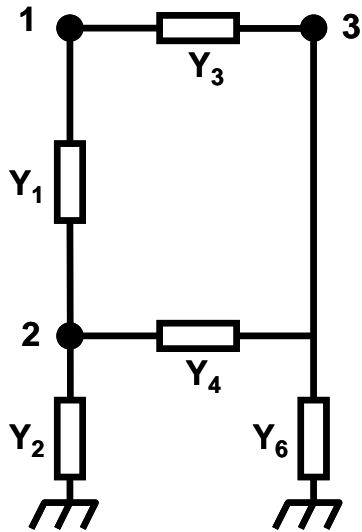
- Now using Cramer's rule.

$$V_s = \frac{I_s \begin{vmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 \end{vmatrix}}{|Y|}$$

- Next, we recognize that the determinant in the numerator is equal to the determinant of the original network with nodes 3 and 4 shorted (i.e. dependent source is shorted)...

## Relationship Between Network Determinants and Admittance

- Original network with nodes 3 & 4 shorted



$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

- Therefore the admittance,

$$A = \frac{I_s}{V_s} = \frac{|Y|}{|Y|_{(V_3 \& V_4 \text{ shorted})}}$$

- Note that this shorted network is passive (no RHP zeroes), but is not of the same order as the original network.

## Relationship Between Network Determinants and Admittance

- Next, we calculate admittance (at the same nodes) from a known stable operating point (gm=0 for example).

$$A_0 = \frac{I_s}{V_s} = \frac{|Y_{stable}|}{|Y_{stable}|_{(V_3 \& V_4 \text{shorted})}} \quad A = \frac{I_s}{V_s} = \frac{|Y|}{|Y|_{(V_3 \& V_4 \text{shorted})}}$$

- Since,  $|Y_{stable}|_{(V_3 \& V_4 \text{shorted})} = |Y|_{(V_3 \& V_4 \text{shorted})}$

- Because the dependent source has been shorted.

$$NDF = \frac{A}{A_0} = \frac{|Y|}{|Y_{stable}|}$$

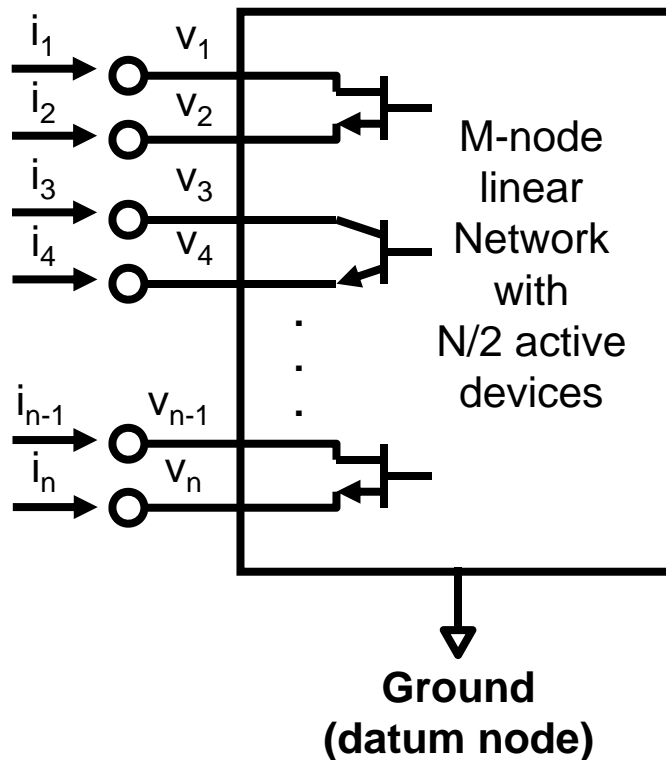
## Appendix 5

# Rigorous Linear Stability Analysis

Wayne Struble

## Rigorous Linear Stability Analysis (networks with many dependent sources)

- Consider the network with many dependent sources (i.e. transistors) and matrix equation shown below.

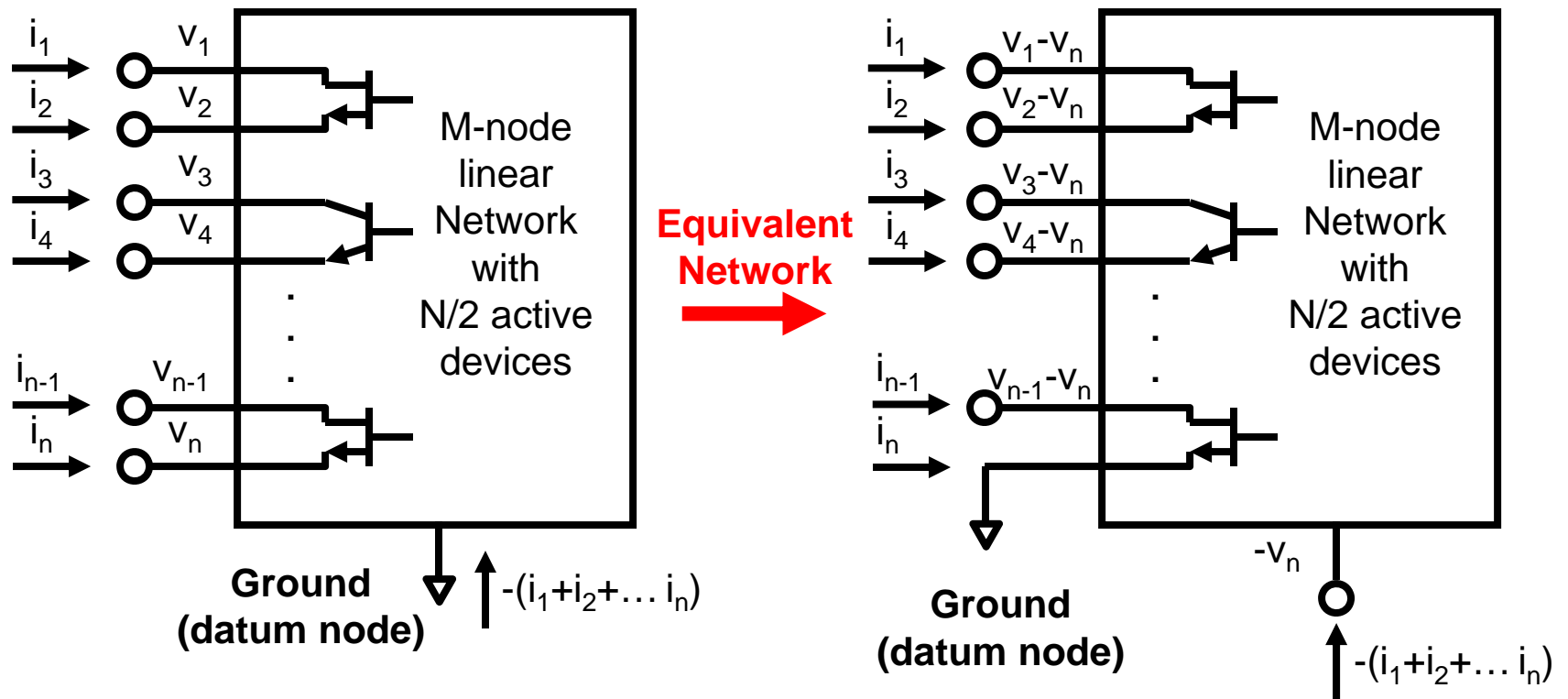


The perturbation nodes are chosen at the outputs of all active devices

$$\begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix}$$

# Rigorous Linear Stability Analysis

- From network theory, any node contained within a network can be designated as the datum node (ground node) without affecting the network (and thus the network determinant).



# Rigorous Linear Stability Analysis

- The indefinite [Y] matrix is defined as:

$$\left[ \begin{array}{c|ccc} \sum_{i=1}^n y_{ij} & -\sum_{i=1}^n y_{i1} & \cdots & -\sum_{i=1}^n y_{in} \\ \hline -\sum_{j=1}^n y_{1j} & y_{11} & \cdots & y_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ -\sum_{j=1}^n y_{nj} & y_{n1} & \cdots & y_{nn} \end{array} \right] \begin{bmatrix} 0 \\ v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} -\sum_{k=1}^n i_k \\ i_1 \\ \vdots \\ i_n \end{bmatrix}$$

- Note that the datum node (ground) contribution to the indefinite network matrix is a linear summation of rows and/or columns of the definite [Y] matrix.

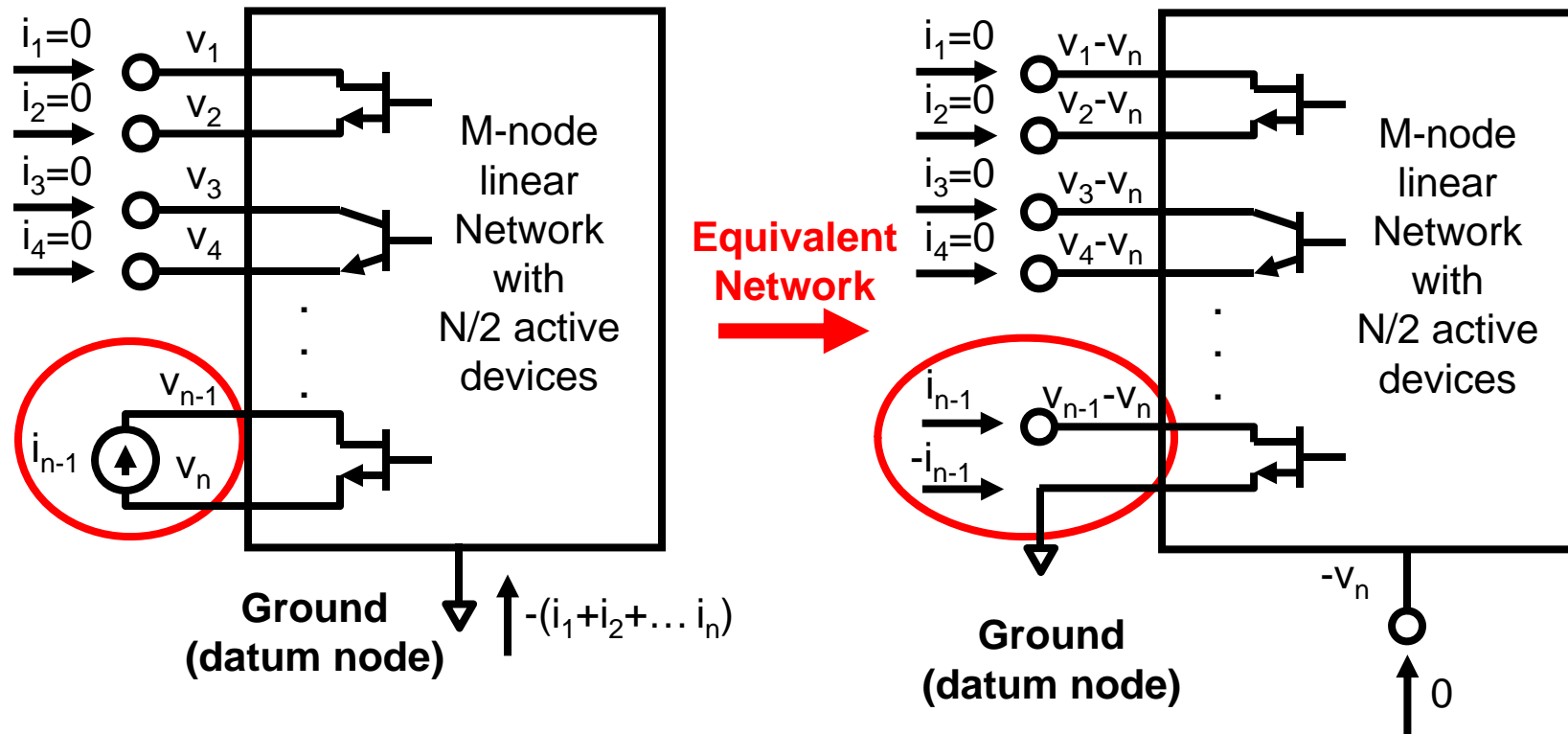
# From The Properties of Determinants

- 1. Switching two rows or columns changes the sign.
- 2. Scalars can be factored out from rows and columns.
- 3. Multiples of rows and columns can be added together without changing the determinant's value.
- 4. Scalar multiplication of a row by a constant  $C$  multiplies the determinant by  $C$ .
- 5. A determinant with a row or column of zeros has value 0.
- 6. Any determinant with two rows or columns equal has value 0.



# Rigorous Linear Stability Analysis

- So, if one stimulates nodes  $n$  and  $n-1$  with external perturbation current source  $i_{n-1}$  (and all other node current stimulations are zero), we get...



# Rigorous Linear Stability Analysis

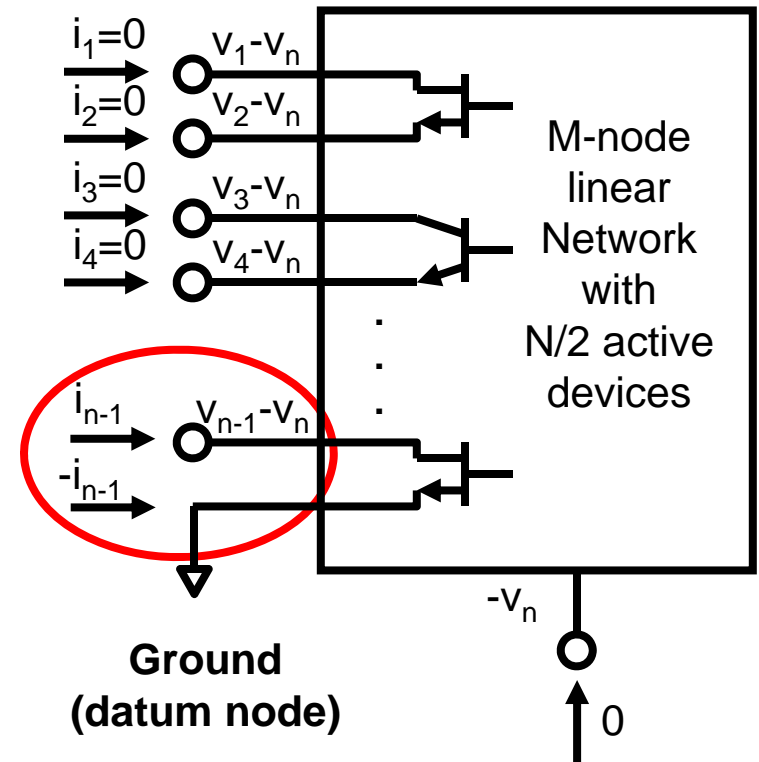
- The network [Y] matrix equations are:

**Indefinite [Y] matrix**

$$\begin{bmatrix} \sum_{i=1}^n y_{ij} & -\sum_{i=1}^n y_{i1} & \cdots & -\sum_{i=1}^n y_{i,n-1} & -\sum_{i=1}^n y_{i,n} \\ -\sum_{j=1}^n y_{1j} & y_{11} & \cdots & y_{1,n-1} & y_{1,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\sum_{j=1}^n y_{n-1,j} & y_{n-1,1} & \cdots & y_{n-1,n-1} & y_{n-1,n} \\ -\sum_{j=1}^n y_{nj} & y_{n1} & \cdots & y_{n,n-1} & y_{nn} \end{bmatrix} \begin{bmatrix} -v_n \\ (v_1 - v_n) \\ \vdots \\ (v_{n-1} - v_n) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ i_{n-1} \\ -i_{n-1} \end{bmatrix}$$

**definite [Y] matrix**

$$\begin{bmatrix} \sum_{i=1}^n y_{ij} & -\sum_{i=1}^n y_{i1} & \cdots & -\sum_{i=1}^n y_{i,n-1} \\ -\sum_{j=1}^n y_{1j} & y_{11} & \cdots & y_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ -\sum_{j=1}^n y_{n-1,j} & y_{n-1,1} & \cdots & y_{n-1,n-1} \end{bmatrix} \begin{bmatrix} -v_n \\ (v_1 - v_n) \\ \vdots \\ (v_{n-1} - v_n) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ i_{n-1} \end{bmatrix}$$



# Rigorous Linear Stability Analysis

- Partition the definite [Y] matrix as follows:

$$\left[ \begin{array}{ccc|c} \sum_{i=1, j=1}^n y_{ij} & -\sum_{i=1}^n y_{i1} & \cdots & -\sum_{i=1}^n y_{i, n-1} \\ -\sum_{j=1}^n y_{1j} & y_{11} & \cdots & y_{1, n-1} \\ \vdots & \vdots & \ddots & \vdots \\ -\sum_{j=1}^n y_{n-1, j} & y_{n-1, 1} & \cdots & y_{n-1, n-1} \end{array} \right] = \left[ \begin{array}{c|c} A_N & B_N \\ \hline C_N & D_N \end{array} \right]$$

- Where:

$$[A_N] = \left[ \begin{array}{ccc|c} \sum_{i=1, j=1}^n y_{ij} & -\sum_{i=1}^n y_{i1} & \cdots & -\sum_{i=1}^n y_{i, n-2} \\ -\sum_{j=1}^n y_{1j} & y_{11} & \cdots & y_{1, n-2} \\ \vdots & \vdots & \ddots & \vdots \\ -\sum_{j=1}^n y_{n-2, j} & y_{n-2, 1} & \cdots & y_{n-2, n-2} \end{array} \right] \quad [B_N] = \left[ \begin{array}{c} -\sum_{i=1}^n y_{i, n-1} \\ y_{1, n-1} \\ \vdots \\ y_{n-2, n-1} \end{array} \right]$$

$$[C_N] = \left[ \begin{array}{ccc|c} -\sum_{j=1}^n y_{n-1, j} & y_{n-1, 1} & \cdots & y_{n-1, n-2} \end{array} \right] \quad [D_N] = [y_{n-1, n-1}]$$

# Rigorous Linear Stability Analysis

- Now, solving for the admittance  $i_{n-1}/(v_{n-1}-v_n)$ :

$$\left[ \begin{array}{c|c} A_N & B_N \\ \hline C_N & D_N \end{array} \right] \left[ \begin{array}{c} V_1 \\ V_2 \end{array} \right] = \left[ \begin{array}{c} 0 \\ I_2 \end{array} \right] \quad \text{Where:} \quad \left[ V_1 \right] = \begin{bmatrix} -v_n \\ (v_1 - v_n) \\ \vdots \\ (v_{n-2} - v_n) \end{bmatrix} \quad \left[ V_2 \right] = [(v_{n-1} - v_n)]$$

$$\left[ I_2 \right] = [i_{n-1}]$$

- Expanding gives:

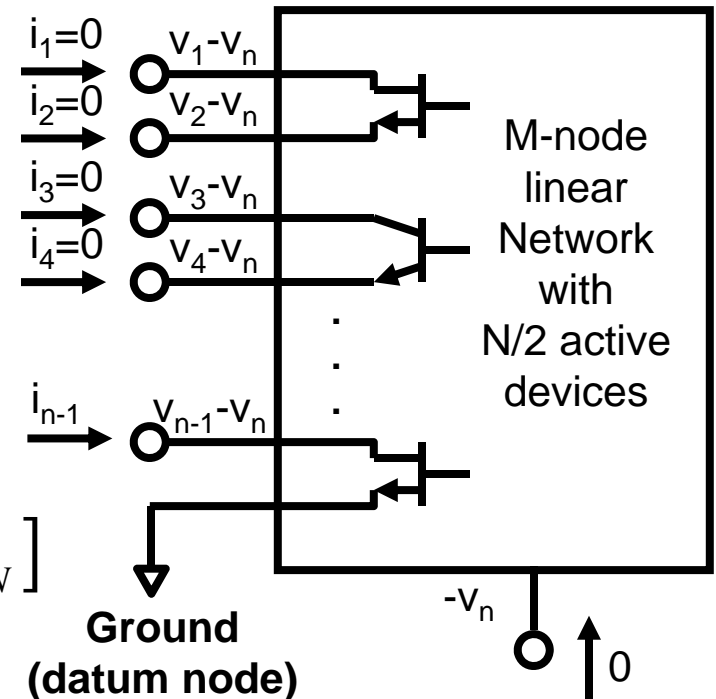
$$\left[ A_N \right] \left[ V_1 \right] + \left[ B_N \right] \left[ V_2 \right] = [0]$$

$$\left[ C_N \right] \left[ V_1 \right] + \left[ D_N \right] \left[ V_2 \right] = [I_2]$$

- Solve for  $[V_1]$  and substitute:

$$\left[ V_1 \right] = -\left[ A_N \right]^{-1} \left[ B_N \right] \left[ V_2 \right]$$

$$\frac{\left[ I_2 \right]}{\left[ V_2 \right]} = \frac{i_{n-1}}{(v_{n-1} - v_n)} = \left[ D_N \right] - \left[ C_N \right] \left[ A_N \right]^{-1} \left[ B_N \right]$$



# Rigorous Linear Stability Analysis

- The result,  $[D_N] - [C_N][A_N]^{-1}[B_N]$  is a scalar (1x1 matrix) so the determinant is equal to the value of the scalar.

$$\frac{i_{n-1}}{(v_{n-1} - v_n)} = [D_N] - [C_N][A_N]^{-1}[B_N]$$

- Therefore:

$$\frac{i_{n-1}}{(v_{n-1} - v_n)} = |D_N - C_N A_N^{-1} B_N|$$

- From the theory of determinants of a subdivided matrix ref [10]:

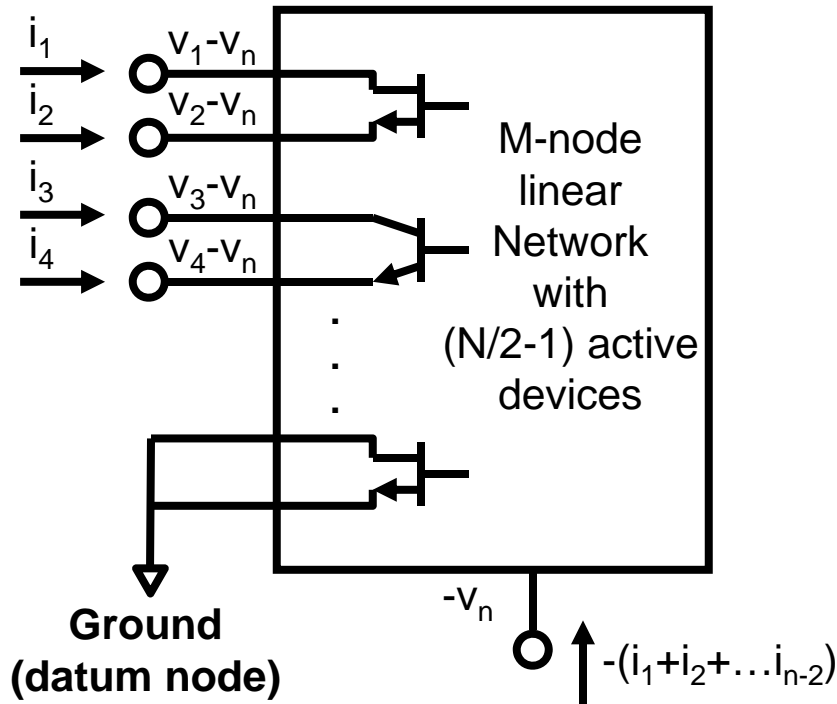
$$|Y| = \left| \begin{array}{c|c} A & B \\ \hline C & D \end{array} \right| = |A - BD^{-1}C| \cdot |D| = |D - CA^{-1}B| \cdot |A|$$

- The admittance below is related to the full network determinant:

$$\frac{i_{n-1}}{(v_{n-1} - v_n)} = a_1 = \frac{|Y|}{|A_N|} \quad \text{Where } |A_N| \text{ may have RHP zeroes due to other active transistors resulting in pole-zero cancellation in } |Y|/|A_N|.$$

# Rigorous Linear Stability Analysis

- However, if we now short circuit the first perturbed nodes (at AC frequencies only, so the transistor biases are unaffected) we get...



Partition the Network Matrix as:

$$\left[ \begin{array}{c|c} A_N & B_N \\ \hline C_N & D_N \end{array} \right] \begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[A_N][V_1] + [B_N][0] = [I_1]$$

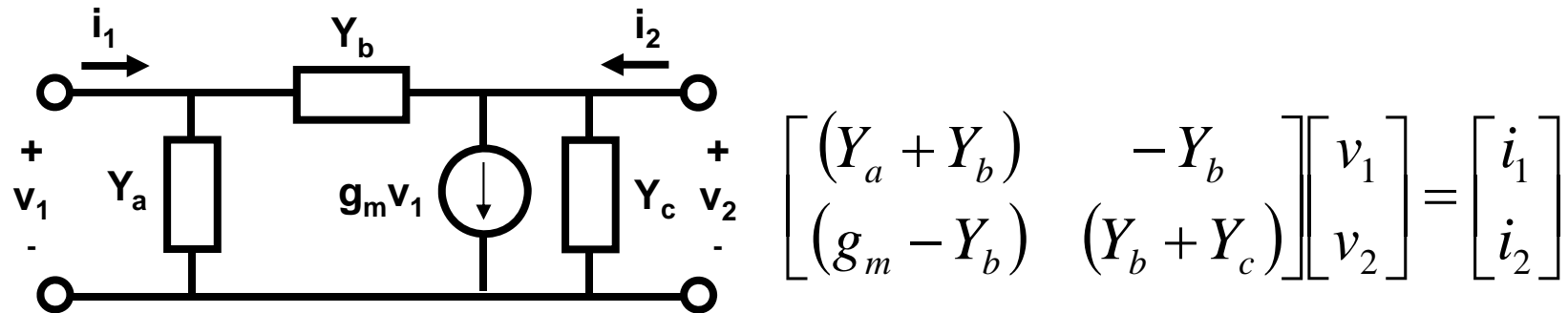
$$[C_N][V_1] + [D_N][0] = [I_2]$$

- This isolates, but does not change, the sub-network  $[A_N]$  from the full network  $[Y]$ , in affect, creating a new network  $[A_N]$  that is of order 1 smaller than  $[Y]$ .

$$[A_N][V] = [I]$$

# Rigorous Linear Stability Analysis

- Remember the simplified network representation of an active transistor (ala D.J.H. Maclean [4]).

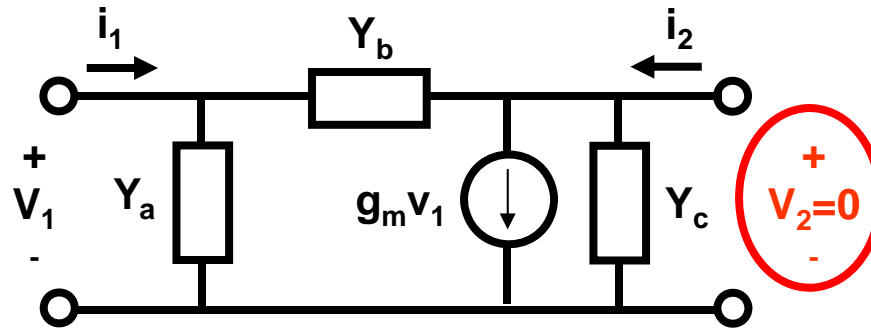


$$(Y_a + Y_b)v_1 - Y_b v_2 = i_1$$

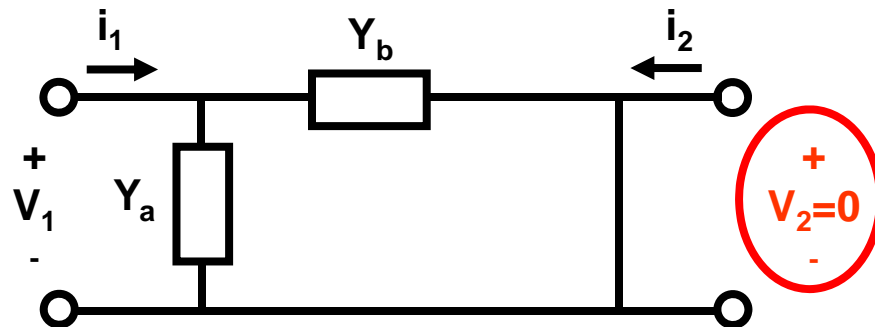
$$(g_m - Y_b)v_1 + (Y_b + Y_c)v_2 = i_2$$

# Rigorous Linear Stability Analysis

- By shorting the dependent source of the active transistor model at AC frequencies only ( $v_2=0$ ), we have made the transistor passive so it can no longer contribute any RHP zeroes to the remaining network.



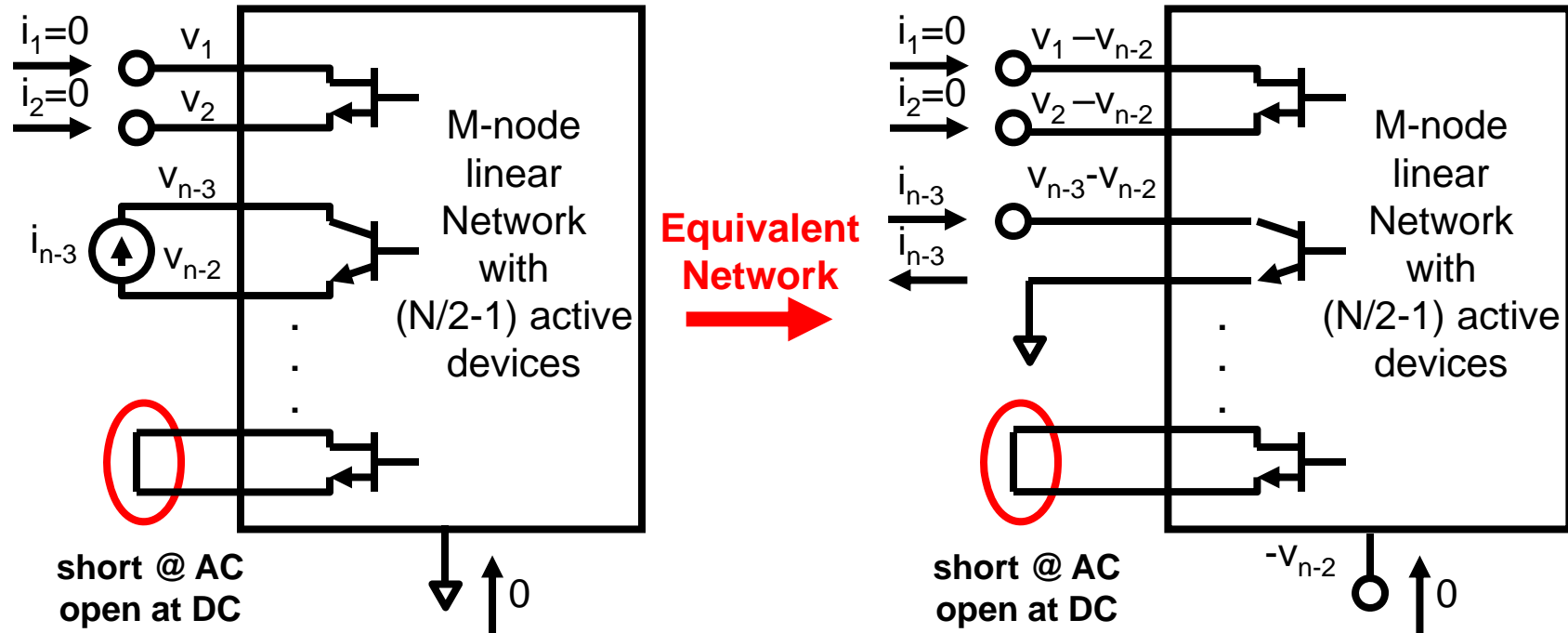
Equivalent network @  
AC frequencies





# Rigorous Linear Stability Analysis

- Next, we simply repeat the procedure (perturb at a second active transistor contained in sub-network  $[A_N]$  with the first active transistor shorted at AC frequencies)...



# Rigorous Linear Stability Analysis

- Partition the remaining  $[A_N]$  network matrix as:

$$[A_N][V] = [I]$$

$$\left[ \begin{array}{c|c} A_{N-1} & B_{N-1} \\ \hline C_{N-1} & D_{N-1} \end{array} \right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}$$

**Where:**

$$[V_1] = \begin{bmatrix} -v_{n-2} \\ (v_1 - v_{n-2}) \\ \vdots \\ (v_{n-4} - v_{n-2}) \end{bmatrix} \quad [V_2] = [(v_{n-3} - v_{n-2})]$$

$$[I_2] = [i_{n-3}]$$

- Solving for the admittance  $[I_2]/[V_2]$ :

$$\begin{aligned} [A_{N-1}][V_1] + [B_{N-1}][V_2] &= [0] \\ [C_{N-1}][V_1] + [D_{N-1}][V_2] &= [I_2] \end{aligned}$$

**Results in:**

$$\frac{i_{n-3}}{(v_{n-3} - v_{n-2})} = a_2 = \frac{|A_N|}{|A_{N-1}|}$$

- Multiplying this admittance result by the previous one we get:

$$a_1 \cdot a_2 = \frac{|Y|}{\cancel{|A_N|}} \cdot \frac{\cancel{|A_N|}}{|A_{N-1}|} = \frac{|Y|}{|A_{N-1}|}$$

# Rigorous Linear Stability Analysis

- If we continue to repeat the procedure until we have perturbed all active transistors contained in network  $[Y]$  (and short circuited all subsequent perturbed nodes at AC frequencies)...

$$a_1 \cdot a_2 \cdots a_N = \frac{|Y|}{|A_1|}$$

- $|A_1|$  contains no RHP zeroes (all active transistors have been shorted), so  $|Y|/|A_1|$  cannot have any pole-zero cancellations.
- Now, repeat the process for the same linear network from a known stable operating point (for normalization) and we get...

$$a_{01} \cdot a_{02} \cdots a_{0N} = \frac{|Y_{stable}|}{|A_1|}$$

# Rigorous Linear Stability Analysis

- Use this result to normalize and we get the Normalized Determinant Function.

$$NDF = \frac{|Y|}{|Y_{stable}|} = \prod_{i=1}^N \frac{a_i}{a_{0i}}$$

- Where N is the number of active transistors in the linear network,  $a_i$  is the measured admittance of the  $i^{\text{th}}$  active transistor due to a perturbing current with all previous transistors shorted at AC frequencies, and  $a_{0i}$  is the measured admittance of the same transistor due to the same perturbing current with all previous transistors shorted at AC frequencies, where the network is operating from a known stable state.

# References

- [1] E. Routh, "Dynamics of a System of Rigid Bodies", 3rd Ed., Macmillan, London, 1877
- [2] H. Nyquist, "Regeneration Theory", Bell System Technical Journal, Vol. 11, pp. 126-147, Jan. 1932
- [3] H. Bode, "Network Analysis and Feedback Amplifier Design", D. Van Nostrand Co. Inc., New York, 1945
- [4] D.J.H. Maclean, "Broadband Feedback Amplifiers", Research Studies Press, 1982
- [5] A. Platzker, W. Struble, and K. Hetzler, "Instabilities Diagnosis and the Role of K in Microwave Circuits", IEEE MTT-S Digest, vol. 3, pp. 1185-1188, Jun. 1993
- [6] W. Struble and A. Platzker, "A Rigorous Yet Simple Method For Determining Stability of Linear N-port Networks", 15th Annual GaAs IC Symposium Digest, pp. 251-254, Oct. 1993
- [7] A. Platzker and W. Struble, "Rigorous Determination of The Stability of Linear N-node Circuits From Network Determinants and The Appropriate Role of The Stability Factor K of Their Reduced Two-Ports", 3rd International Workshop on Integrated Nonlinear Microwave and Millimeterwave Circuits, pp. 93-107, Oct. 1994
- [8] J. Jugo, J. Portilla, A. Anakabe, A. Suarez and J.M. Collantes, "Closed-Loop Stability Analysis of Microwave Amplifiers", Electronics Letters, vol.37 No. 4, pp. 226-228, Feb. 2001
- [9] C. Barquinero, A. Suarez, A. Herrera and J.L. Garcia, "Complete Stability Analysis of Multifunction MMIC Circuits", IEEE Tran. Microwave Theory and Techniques, vol.55 No. 10, pp. 2024-2033, Oct. 2007
- [10] I. Schur, "Ueber Potenzreihen die im Innern des Einheitskreises beschraenkt sind", Journal fuer Reine und Angewandte Mathematik, 1917, 147, 205-232.