

## CIRCUIT ANALYSIS USING NETWORK PARAMETERS

We all want to understand and predict how a circuit works, so we want to be able to represent circuit behavior by using mathematical expressions or equations. The starting place is linearity. If a circuit consists of linear elements, then it can be represented by a set of linear equations relating its independent quantities, like currents and voltages, to its dependent quantities, also currents and voltages. This remarkable fact allows us to take a complicated circuit with lots of pieces, put it into a black box, pull out two pairs of nodes defined by voltages and currents, and reduce it all to two linear equations relating these voltages and currents as shown below.

$$F_1 = m_{11} * X_1 + m_{12} * X_2$$

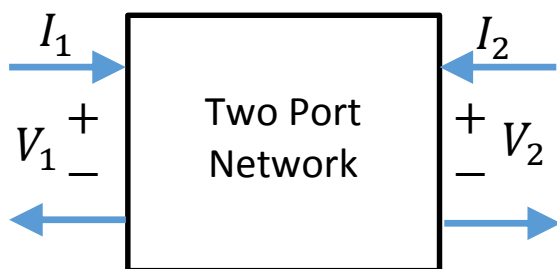
$$F_2 = m_{21} * X_1 + m_{22} * X_2$$

This makes intuitive sense since the circuit performance is the sum of the performance of a bunch of linear sub-circuits. Network parameters are the backbone of all those CAD programs you use to do circuit design. All those measurement programs to test and characterize your designs translate the data into network parameter files (usually S-parameters) used to store, display and analyze your work. Transmission lines are also included in the list of circuit elements since the Telegrapher's equations show that a transmission line can be characterized as a set lumped element circuits specified at a single frequency and for a specific characteristic impedance, length and propagation constant.

We are free to choose what circuit quantities we wish for the  $F$  and  $X$  variables of our linear equations. This leads to many different combinations defining different network parameter sets such as Y-parameters, Z-parameters, h-parameters, ABCD-parameters and, with a switch from voltages and currents to reflected power and transmitted power, to S-parameters and T-parameters. It is your choice as to which one makes your design easier, but remember that standard conventions apply to the terminal definitions of the currents and voltages for consistency. Note that if the quantities are conveniently measurable, we can determine the  $m$  coefficients in our equations by measurements.

$$m_{11} = \left. \frac{F_1}{X_1} \right|_{X_2=0} \quad m_{12} = \left. \frac{F_1}{X_2} \right|_{X_1=0}$$
$$m_{21} = \left. \frac{F_2}{X_1} \right|_{X_2=0} \quad m_{22} = \left. \frac{F_2}{X_2} \right|_{X_1=0}$$

Here are the definitions of some of the most common network parameter sets.



### Y-Parameters

$$I_1 = y_{11} * V_1 + y_{12} * V_2$$

$$I_2 = y_{21} * V_1 + y_{22} * V_2$$

### Z-Parameters

$$V_1 = z_{11} * I_1 + z_{12} * I_2$$

$$V_2 = z_{21} * I_1 + z_{22} * I_2$$

### h-Parameters

$$V_1 = z_{11} * I_1 + z_{12} * V_2$$

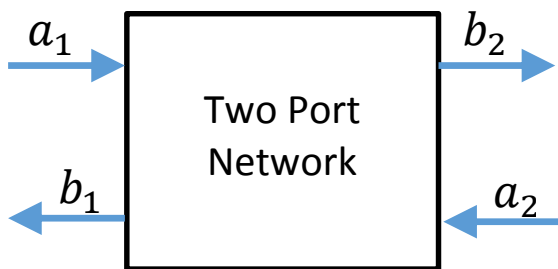
$$I_2 = z_{21} * I_1 + z_{22} * V_2$$

### ABCD-Parameters

$$V_1 = A * V_2 + B * (-I_2)$$

$$I_1 = C * V_2 + D * (-I_2)$$

Here are the definitions of some of the power-based network parameter sets.



#### S-Parameters

$$b_1 = S_{11} * a_1 + S_{12} * a_2$$

$$b_2 = S_{21} * a_1 + S_{22} * a_2$$

#### T-Parameters

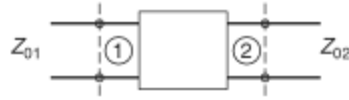
$$b_1 = T_{11} * a_2 + T_{12} * b_2$$

$$a_1 = T_{21} * a_2 + T_{22} * b_2$$

#### Some Final Comments and Cautions

1. For some two port networks, the Z- and Y- parameters are not defined and do not exist because it is impossible to express the voltages in terms of the currents, and vice versa. Thus, the ideal transformer has no Z- or Y- parameters, since the equations for an ideal transformer are  $V_1 = \frac{1}{n}V_2$  and  $I_1 = -nI_2$ .
2. Hybrid (h) parameters make  $I_1$  and  $V_2$  dependent variables and can be used for ideal transformers, but their main claim to fame is to provide equations that characterize the performance of transistors.
3. Series interconnections of Z-parameter networks are simple to compute but may lead to errors. If you add two Z parameter network matrices  $[Z_a] + [Z_b]$ , the result  $[Z_{series}]$  is the Z-parameter matrix of the two networks connected in series. Caution is needed when using network parameter matrix additions and multiplications to obtain the parameters of interconnected networks. If the interconnections create a new circuit where the current in one terminal of a port does not equal the current of the other terminal of the same port, then the resulting parameter matrix will not faithfully represent the combined network. A similar problem may occur in combining Y parameter networks interconnected in parallel.
4. Cascade interconnections are easy with ABCD-parameters or T-parameters since the parameters of the resulting cascaded network are simply the result of multiplying the individual network parameter matrices together:  
 $[T_{cascade}] = [T_a] * [T_b]$  and  $[ABCD_{cascade}] = [ABCD_a] * [ABCD_b]$ .
5. Network parameters are ratios network quantities, like voltages and currents or reflected and transmitted power, so if the networks have resonances or other behavior that result in the denominator of the network parameter computation becoming vanishingly small, the resulting network parameters accuracy can be compromised. i.e. What happens to  $S_{21} = \frac{b_2}{a_1} \Big|_{a_2=0}$  if  $a_1 \cong 0$ .
6. Finally, if you are curious about the  $(-I_2)$  term in the ABCD parameters, it is because Lord Kelvin (yes, him) decided to use the current exiting the port terminal as the dependent variable in opposition to the definition that has superseded it. The minus sign in the ABCD definitions forces the direction of the current to be consistent with other network parameter definitions (e.g. Z, Y, and H).
7. The table below was provided with permission from Inder J. Bahl and it lists formulas that allow you to convert from one set of network parameters to a different set of network parameters. Mongia, R. K., Bahl, Inder J., et. al., *RF and Microwave Coupled-Line Circuits*, 2<sup>nd</sup> ed, Norwood, MA: Artech House, 2007, pg 43.

## Conversion Equations between Various Representative Two-Port Network Parameters



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{C} \begin{bmatrix} A & AD - BC \\ 1 & D \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{B} \begin{bmatrix} D & -(AD - BC) \\ -1 & A \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{Z_{21}} \begin{bmatrix} Z_{11} & (Z_{11}Z_{22} - Z_{12}Z_{21}) \\ 1 & Z_{22} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{-Y_{21}} \begin{bmatrix} Y_{22} & 1 \\ Y_{11}Y_{22} - Y_{12}Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{\left(\frac{Z_{11}}{Z_{01}} + 1\right)\left(\frac{Z_{22}}{Z_{02}} + 1\right) - \frac{Z_{12}Z_{21}}{Z_{01}Z_{02}}} \times \begin{bmatrix} \left(\frac{Z_{11}}{Z_{01}} - 1\right)\left(\frac{Z_{22}}{Z_{02}} + 1\right) - \frac{Z_{12}Z_{21}}{Z_{01}Z_{02}} & 2\frac{Z_{12}}{Z_{02}} \\ 2\frac{Z_{21}}{Z_{01}} & \left(\frac{Z_{11}}{Z_{01}} + 1\right)\left(\frac{Z_{22}}{Z_{02}} - 1\right) - \frac{Z_{12}Z_{21}}{Z_{01}Z_{02}} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{\left(1 + \frac{Y_{11}}{Y_{01}}\right)\left(1 + \frac{Y_{22}}{Y_{02}}\right) - \frac{Y_{12}Y_{21}}{Y_{01}Y_{02}}} \times \begin{bmatrix} \left(1 - \frac{Y_{11}}{Y_{01}}\right)\left(1 + \frac{Y_{22}}{Y_{02}}\right) + \frac{Y_{12}Y_{21}}{Y_{01}Y_{02}} & -2\frac{Y_{12}}{Y_{01}} \\ -2\frac{Y_{21}}{Y_{02}} & \left(1 + \frac{Y_{11}}{Y_{01}}\right)\left(1 - \frac{Y_{22}}{Y_{02}}\right) + \frac{Y_{12}Y_{21}}{Y_{01}Y_{02}} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{(1 + S_{11})(1 + S_{22}) - (S_{12}S_{21})} \times \begin{bmatrix} Y_{01}[(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}] & -2Y_{01}S_{12} \\ -2Y_{02}S_{21} & Y_{02}[(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}] \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{(2S_{21})} \begin{bmatrix} [(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}] & Z_{02}[(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}] \\ \frac{1}{Z_{01}}[(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}] & \frac{Z_{01}}{Z_{02}}[(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}] \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{(B + CZ_{01}Z_{02}) + (AZ_{02} + DZ_{02})} \times \begin{bmatrix} (B - CZ_{01}Z_{02}) + (AZ_{02} - DZ_{01}) & 2Z_{01}(AD - BC) \\ (B - CZ_{01}Z_{02}) - (AZ_{02} - DZ_{01}) & \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}} \times \begin{bmatrix} Z_{01}[(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}] & 2Z_{02}S_{12} \\ \frac{2Z_{01}S_{12}}{2Z_{01}S_{21}} & Z_{02}[(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}] \end{bmatrix}$$