

***Stability Analysis for RF and
Microwave Circuit Design***

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Stability in Electrical Circuits

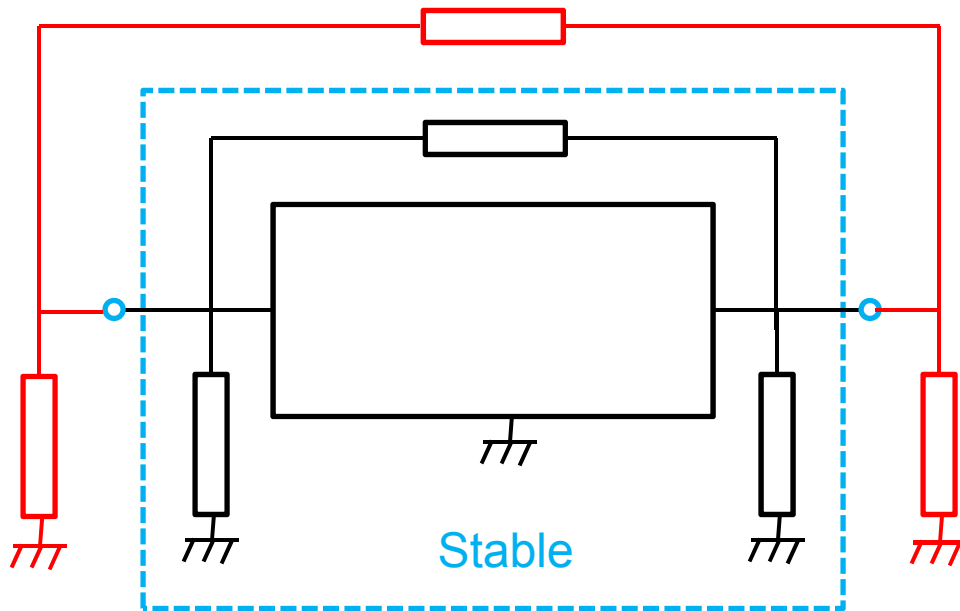
- In an ideal linear system, stability can be defined in several ways:
 - 1) A BIBO (bounded input bounded output) system is stable
 - 2) A system, the response of which its $\delta(t)$ decays to 0 is stable
 - 3) A system which delivers only ω_0 signals in response to ω_0 excitations is stable
- Real circuits are bounded by noise floors at their low levels and nonlinearities at their high levels. The noise floor insures the presence of outputs with no inputs and the nonlinearities may mask instabilities generated by the system by attenuating them.
- These considerations should be taken into account when ascertaining whether a circuit is stable or not in the laboratory.
- In this talk we focus our attention on instabilities in the design phase of the circuits where the detection of instabilities is obvious since it is subject to rigorous mathematical analysis.
- After the talk it will be clear that a circuit with any non negative real parts of its characteristic zeros is unstable.



Historical Background

Where do stability factors come from?

- In the early days of electronic circuits 1930-1960, in large part in Bell Laboratories, but also elsewhere, amplifier circuits were built in the laboratory, and once stabilized, were incorporated in larger circuits, either in cascade or in balanced configurations. Sometimes these larger circuits oscillated. Several researchers, among them, Llewellyn, Linvill, Nyquist, Bode, Black, Stern, Mason, realized that the source of oscillations were circuit poles residing in the RHP (Right Half Plane). Stability factors or criteria, based on laboratory characterizations, were devised to insure that this will not happen.



A 2-Port without additional feedback is potentially unstable if:

$$C = \frac{|K_{12}K_{21}|}{2 \operatorname{Re}(K_{11}) \operatorname{Re}(K_{22}) - \operatorname{Re}(K_{12}K_{21})} > 1$$

Where K_{ij} are either Y or Z parameters

Feedback amplifiers are stable if the zeros of $1 + \mu\beta$ are in the LHP

$$\frac{V_{out}}{V_{in}} = \frac{\mu}{1 + \mu\beta}$$

Where do stability factors come from?

- In the no-additional feedback case, the potential instability of the network was arrived at by noticing that under certain passive terminations, the output power becomes negative, or alternatively, the real part of either the input or output impedance becomes negative. Either approach results in the same criterion.
- In the early 60's first Venkateswaran and then Rollet noticed that the instability criterion now written as the inverse of C (Venkateswaran's η , Rollett's K) is invariant in the Z,Y,H and G matrix parameters and attributed great significance to this fact.

$$K = \frac{2 \operatorname{Re}(\gamma_{11}) \operatorname{Re}(\gamma_{22}) - \operatorname{Re}(\gamma_{12}\gamma_{21})}{|\gamma_{12}\gamma_{21}|} > 1 \quad \operatorname{Re}(\gamma_{11}), \operatorname{Re}(\gamma_{22}) > 0 \text{ where } \gamma_{ij} = \text{either } z_{ij}, y_{ij}, h_{ij}, g_{ij}$$

- Almost in passing, Rollett also introduced a proviso in his paper that warned that the analysis may not be valid in circuits with characteristic frequencies in the RHP.
- This proviso, essentially ignored by modern day designers, effectively says that the stability criteria **are invalid** in all cases where the stability of the “unloaded” circuits is not assured (i.e. when they are unstable).
- **These stability criteria can be applied only to known stable circuits!!**

Where do stability factors come from?


- Rollett's proviso is automatically fulfilled in all circuits where the parameters (Y,Z etc) **are measured**, not calculated. If properly done, the measurement assures the stability of a circuit since an unstable circuit cannot be measured and characterized with the application of external steady state signals.
- In the late 60's, a S-parameter formulation was introduced by Kurokawa, Broadway and Hauri which states that: for absolute stability, two conditions must apply:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |S_{11}S_{22} - S_{12}^* S_{21}|}{2 |S_{12}S_{21}|} > 1 \quad \text{and} \quad |S_{11}S_{22} - S_{12}S_{21}| < 0$$

- The 2nd condition can be expressed in many different ways, the above is just one of them.
- **No new insight or information is gained by the more recent introduction of single stability parameter “ μ ” in place of the two conditions stated before.** In practice $K > 1$ is taken by the vast microwave community as the condition for absolute stability since $|S_{11}S_{22} - S_{12}S_{21}|$ is almost always less than 1.

Where do stability factors come from?

- In the case of $K < 1$, oscillation is not assured unless the proper reactance is introduced. Nature is mischievous, is the current attitude, so stay away from this region. **However, perfectly stable circuits with $K < 1$ can be designed.**
- The above discussion explains why control engineers, oscillator, and feedback amplifier designers do not use the stability criterion K !
- **Circuit designers should not use it also as their only criterion since it does not insure against instabilities not introduced by varying the external terminations (i.e. instabilities inherent in the circuit).**



Rigorous Linear Network Stability Theory

Rigorous Linear Network Stability Theory

- First, forget everything you learned about the popular stability factor k .
- Second, re-read the previous sentence!!
- OK, now that that has sunk in...

- A separate test is required (like the Normalized Determinant Function) to assure the stability of a network before the Linvill or Rollett stability criteria can be applied.
- The NDF technique [5] looks for zeroes in the right half plane (RHP) of the full network determinant by plotting the trajectory versus frequency of the properly normalized linear network determinant.
- Once network stability is assured (**including all feedback paths**), then the C or K factor can be used to determine under which port impedances network stability is maintained.
- Next, we will show how network stability is fundamentally determined from the dynamic response of a network (and that relationship to the full network determinant).

Rigorous Linear Network Stability Theory

- The dynamic response of a linear network can be derived from a set of vector equations whose transform is represented by a matrix equation. For example, the Y (admittance) network representation is:

$$[Y(s)][V(s)] = [I(s)] \quad s = \sigma + j\omega$$

- The general solution $[I]$ of the network subject to any particular steady excitation $[V]$, is composed of a linear superposition of the transient and the steady state responses.
- The transient response is determined by the roots (poles) of the network which are the zeroes of its network determinant $|Y(s)|$.
- The transient response takes the form:

$$\sum_{k=1}^p a_k t^{(m_k-1)} e^{(\sigma_k \pm j\omega_k)t}$$

- where,

$$\sigma_k \pm j\omega_k$$

is the k_{th} root of the network (zero of its determinant) with multiplicity m_k and p is the total number of roots.

Rigorous Linear Network Stability Theory

- Notice that the roots always appear in complex conjugate pairs since the network response is a real function of time.
- By direct inspection of the transient response, we can see that it will die out in time, allowing the system to reach its steady state, if and only if, all $\sigma_k < 0$.

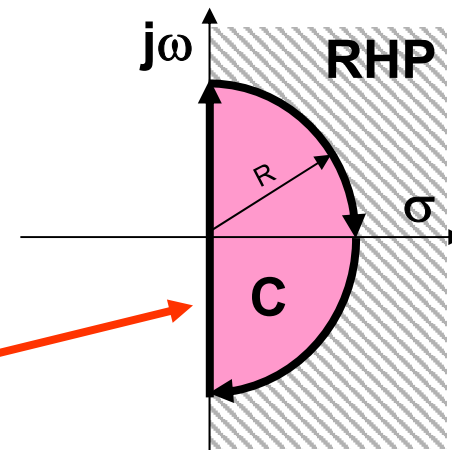
$$\sum_{k=1}^p a_k t^{(m_k-1)} e^{(\sigma_k \pm j\omega_k)t}$$

- Therefore, a linear network is stable, if and only if, all the zeroes of its determinant lie in the left half plane (LHP) provided none of the individual elements have any poles in the RHP. (THIS IS RIGOROUS!)
- This will be the case for all networks composed of elementary elements (L's, C's, R's, Transmission Lines, Dependent Sources such as VCCS, VCVS, CCCS, C CVS, etc.).
- So, how do we determine if the network determinant has any RHP zeroes?

Determination of the Number of RHP Zeroes

- We make use of “The Principle of the Argument Theorem” of complex theory which states that:
- The total change of the argument (phase) of a function $F(s)$ along a closed contour C on which the function has no zeroes and inside which it is analytic except for poles, is equal to:

$$2\pi(N_p - N_z)$$



Note that the contour is clockwise in our nomenclature.

Counterclockwise (std mathematical nomenclature) would give $2\pi(N_z - N_p)$.

- Where N_p is the number of RHP poles and N_z is the number of RHP zeroes of the function $F(s)$ inside C .

Determination of the Number of RHP Zeroes

- To make use of this theorem, we must first normalize the network determinant $|Y(\omega)|$ (to force the function to be finite along the real frequency axis i.e. semicircular contour from $\omega = -\infty$ to $+\infty$).
- We do this by dividing $|Y(\omega)|$ by a second determinant $|Y_0(\omega)|$ **which is of the same rank as $|Y(\omega)|$ and contains no RHP zeroes (i.e. is known stable)**. Therefore, $|Y(\omega)|/|Y_0(\omega)|$ will not have any RHP poles by construction ($N_p=0$).
- We call this function the Normalized Determinant Function or NDF [5] ($NDF(\omega) = |Y(\omega)|/|Y_0(\omega)|$ or Δ/Δ_0).
- The **Normalized Determinant Function will always be finite at infinity, and only contain RHP zeroes from $|Y(s)|$** , so we can use The Principle of the Argument Theorem on NDF to determine if $|Y(s)|$ contains any RHP zeroes N_z (if so, the network is unstable).
- Next, we will describe how to use The Principle of the Argument Theorem on NDF to find RHP zeroes.

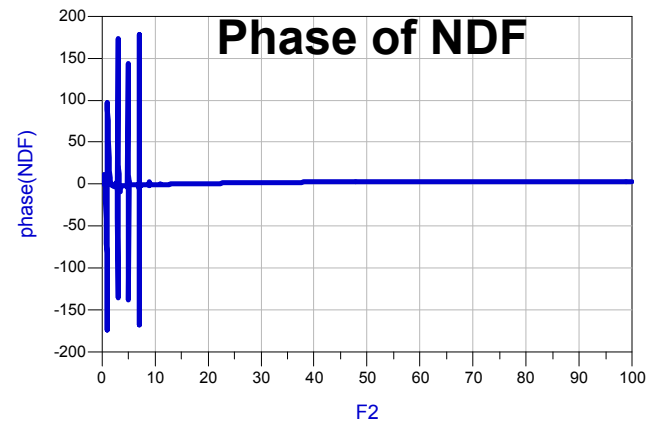
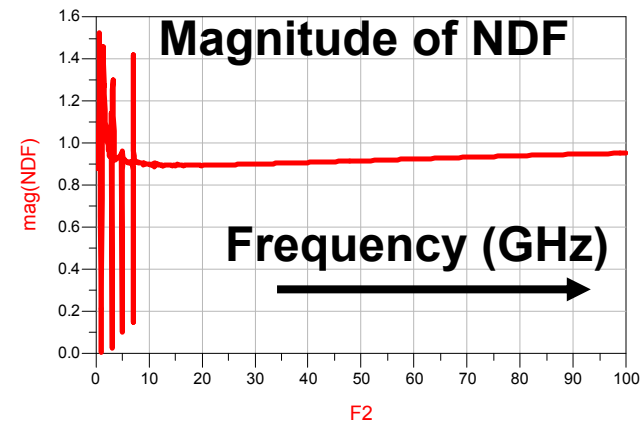
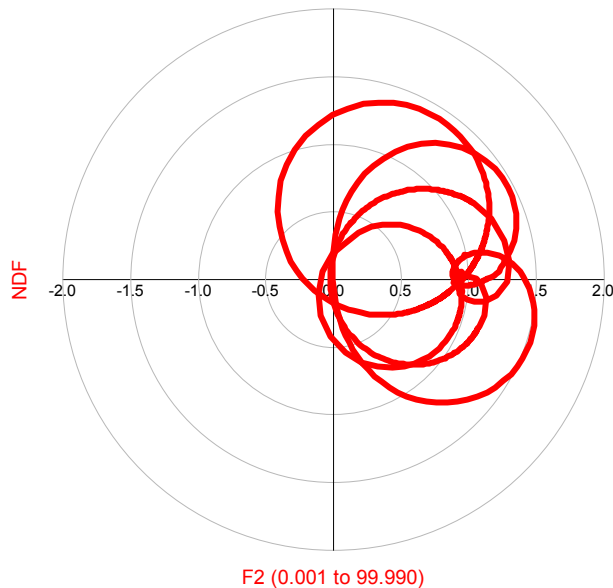
Determination of the Number of RHP Zeroes

- To find RHP zeroes, we plot the complex function $NDF(\omega)$ over real frequencies ω (from $-\infty$ to $+\infty$) and look for clockwise encirclements (N_z) of the origin (0,0) on a polar plot.
- Remember, $N_p=0$ by construction of the NDF, so counter-clockwise encirclements are not possible.
- Every clockwise encirclement of the origin equates to one network determinant zero (of $|Y(s)|$) in the right half plane RHP. (Total clockwise encirclements = N_z)
- **If any RHP zeroes are found in NDF, the linear network $[Y(s)]$ is unstable! (THIS IS RIGOROUS)**
- The converse is also true; **If no RHP zeroes are found in NDF, the linear network $[Y(s)]$ is stable!**
- Zeroes of $|Y(s)|$ come in complex conjugate $\pm j\omega$ pairs (because the network response is a real function of time), so we can plot the trajectory of NDF over positive frequencies only (ω from 0 to $+\infty$) and still capture all stability information.

Determination of the Number of RHP Zeroes

- The polar plot of NDF can be very “messy” and it may be hard to tell if there are clockwise encirclements of the origin.
- The shape of the NDF plot is unimportant, only whether there are clockwise encirclements of the origin.

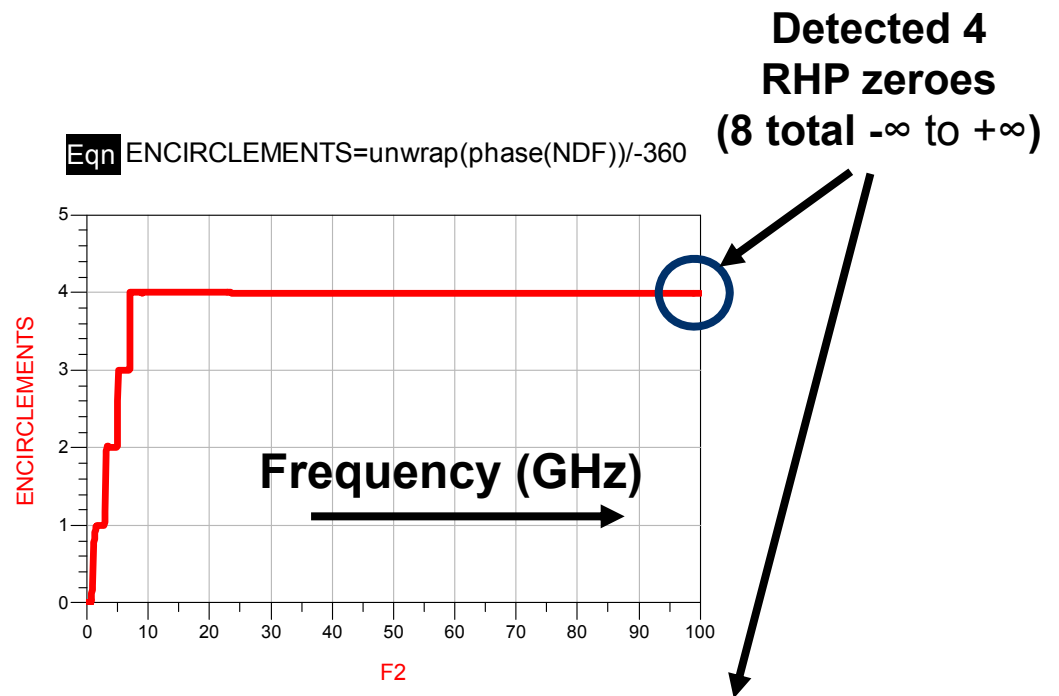
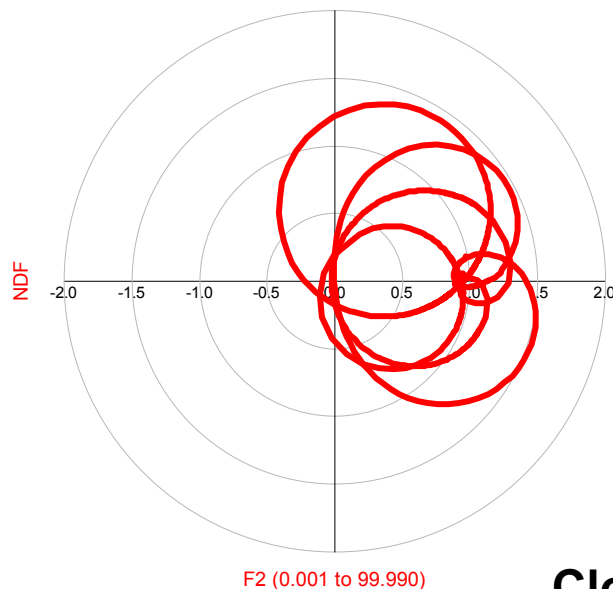
Polar Plot of NDF



Determination of the Number of RHP Zeroes

- To count the number of clockwise encirclements N_z of the origin $(0,0)$, we use a plot of the unraveled phase versus frequency (ω from 0 to $+\infty$) normalized to -360° (-2π radians).
- The resulting phase as one approaches $+\infty$ frequency determines whether RHP zeroes have been detected and the network is unstable.

Polar Plot of NDF



Clockwise Encirclements (0 stable ≥ 1 unstable)

How High in Frequency Do I Need to Calculate NDF?

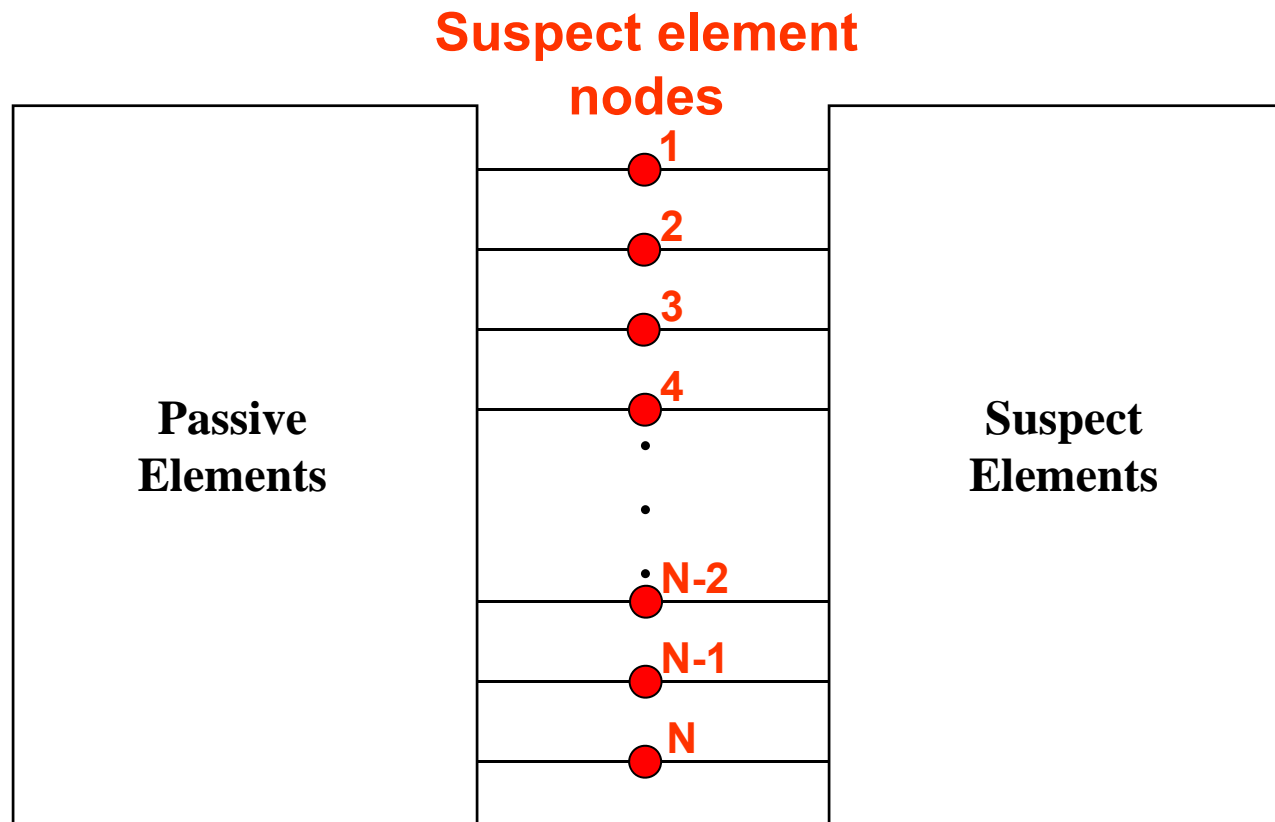
- How High in Frequency Do I Need to Calculate NDF?
- The NDF function will always approach a constant real value (with zero phase) as frequency goes to $+\infty$, so this gives an indication if you have calculated to sufficiently high enough frequencies.
- This depends on the network, but in general one should calculate beyond the highest F_{\max} of the transistors contained in the network.
- **Be careful of [S]-parameter or EM blocks in your network** when calculating NDF because the circuit design software can extrapolate beyond the highest/lowest frequencies in the [S]-parameter file or EM block (and you will get incorrect results).
- If we normalize $|Y(\omega)|$ using the same network with all dependent sources set to zero (as in [5]), then the NDF function will approach (1,0) as frequency goes to $+\infty$. (We will show that there are many other ways to normalize $|Y(\omega)|$ however).
- It is important to calculate the NDF at sufficiently fine frequency steps to generate a smooth contour so that encirclements are not missed.
- We recommend using a logarithmic frequency sweep with many frequency points (100-200) per decade to start your analysis.

OK, how do I calculate NDF for Linear Networks?

- We will show two ways to calculate the NDF function using a ring oscillator example. We will calculate NDF from:
 - 1) Network Determinants directly (preferred closed loop technique)
 - 2) Network Admittances (another closed loop technique)
- There are other options for calculating the NDF such as using Return Ratios (open loop technique [6]). However, we no longer prefer to use these techniques, so I will not discuss them here.
- The first technique is the simplest and only requires two frequency sweeps to calculate the $NDF(\omega)$ function to ascertain stability.
- The second technique requires separate calculations at each suspect element node in the linear network (with subsequent circuit modification), so for a network with N suspect element nodes, one needs to perform a minimum of $2*N$ frequency sweeps to calculate the $NDF(\omega)$ function. (more details to follow...)
- Let's start with NDF from network determinants...

NDF of Linear Networks from Network Determinants

- Every network can be separated into a parallel connection of known passive elements and “suspect” elements.



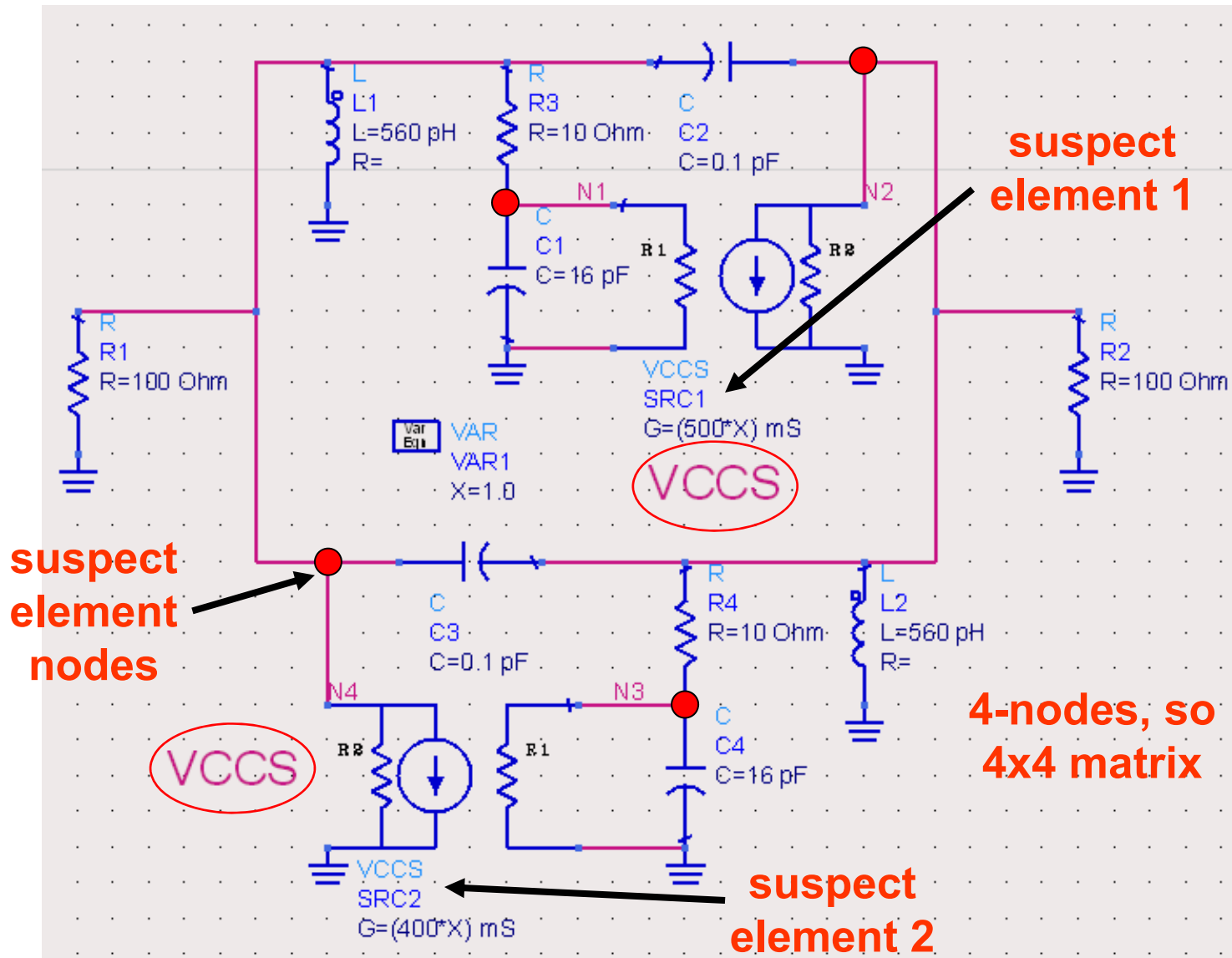
NDF of Linear Networks from Network Determinants

- So what are “suspect” elements?
- Suspect elements are those elements which can cause RHP zeroes to appear in the full network determinant.
- In other words, if all suspect elements are removed from the network, the network is by definition stable (because all remaining elements are passive).
- Suspect elements consist of transistors (including s-parameter files of transistors), dependent sources (VCCS, VCCS, CVCVS, CCCS), and negative valued resistors, inductors, and capacitors (i.e. non-foster elements).
- Suspect elements **DO NOT** consist of positive valued resistors, inductors, capacitors, transmission lines, or any other passive network element (such as transformers, EM blocks, passive s-parameter blocks, etc.).

Practical Techniques for Stability Analysis (NDF using Network Determinants)

- The procedure to calculate the NDF(ω) using network determinants is as follows:
 - 1) First, identify all nodes in the network that are connected to a suspect element. **The network matrix cannot be reduced in size beyond these nodes** (or we will lose rigorous stability information).
 - 2) Then, calculate the determinant of the matrix reduced to these nodes $|Y(\omega)|$. The network matrix can be in either admittance, impedance, or hybrid representation (**NOT S-matrices**).
 - 3) Next, render all suspect elements contained in the network passive and calculate the determinant of this (now) passive network matrix $|Y_0(\omega)|$. To render suspect elements passive, set all dependent sources to zero, multiply the value of any negative R's, L's & C's by -1, and set all transistors (including s-parameter files of transistors) to a known passive state (more on how to do that later).
 - 4) Now, $NDF = |Y(\omega)|/|Y_0(\omega)|$
where: $Y(\omega)$ is the network matrix at frequency ω and $Y_0(\omega)$ is the network matrix with all suspect elements rendered passive.

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Determinants)



Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Determinants)

- To calculate the NDF function, we perform two frequency sweeps (that output Y-parameters) using ADS.
- During the first sweep, we calculate $|Y_0(\omega)|$ by setting the VCCS transconductances “G” to zero (i.e. known passive state).
- During the second sweep, we calculate $|Y(\omega)|$ by setting the VCCS transconductances “G” to their desired (as designed) values.
- Then we simply divide $|Y(\omega)|$ by $|Y_0(\omega)|$ to get $NDF(\omega)$.
- We then plot the $NDF(\omega)$ contour (on a polar plot), and the argument (unraveled phase plot) to see if the contour encircles the origin (0,0) in a clockwise manner.
- If it does, the circuit is unstable! If it does not, the circuit is stable.

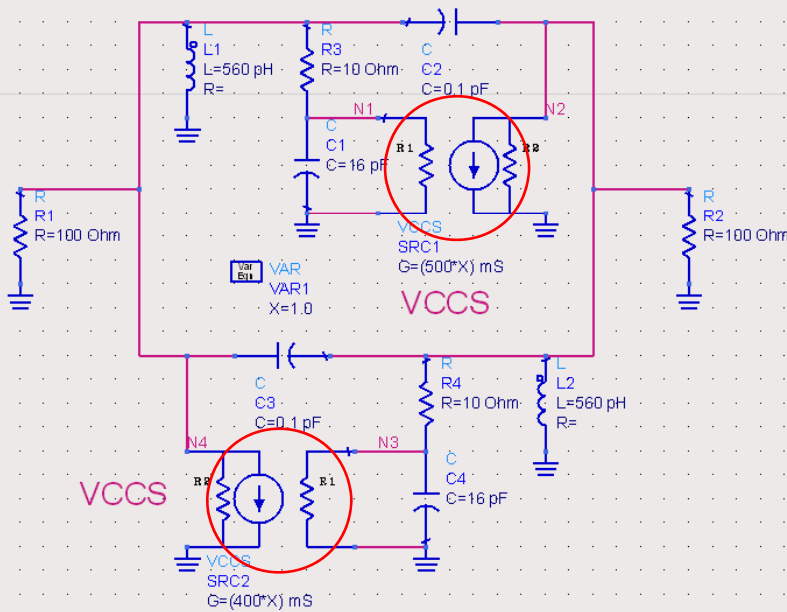
Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Determinants)

ADS NDF Implementation

Stability Analysis of Linear networks $NDF=|Y|/|Y_0|$

Suspect elements are circled (any dependent sources, and -L, -C, -R's)

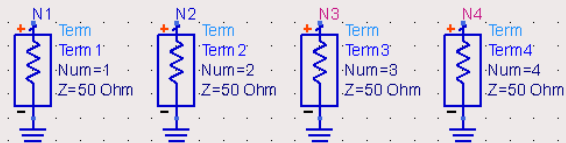
All suspect element nodes must be included in the NDF [Y] and [Y0] matrices



S-PARAMETERS

```
S_Param
SP1
Start=0.001 GHz
Stop=100 GHz
Step=
Dec=500
CalcY=yes
```

S-parameter frequency sweep outputs Y-parameter matrix



PARAMETER SWEEP

```
ParamSweep
Sweep1
SweepVar="X"
Sim InstanceName[1]="SP1"
Sim InstanceName[2]=
Sim InstanceName[3]=
Sim InstanceName[4]=
Sim InstanceName[5]=
Sim InstanceName[6]=
Start=0
Stop=1
Step=1
```

parameter sweep variable X used to toggle between known stable network and network to be analyzed for stability

0-sets dependent sources to zero and -L, -C, -R values to +L, +C, +R values (known stable network)
1-sets all values to normal (network for stability test)

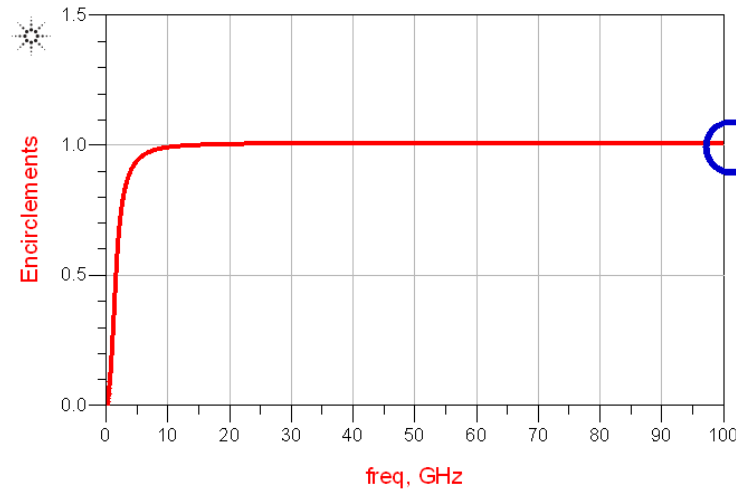
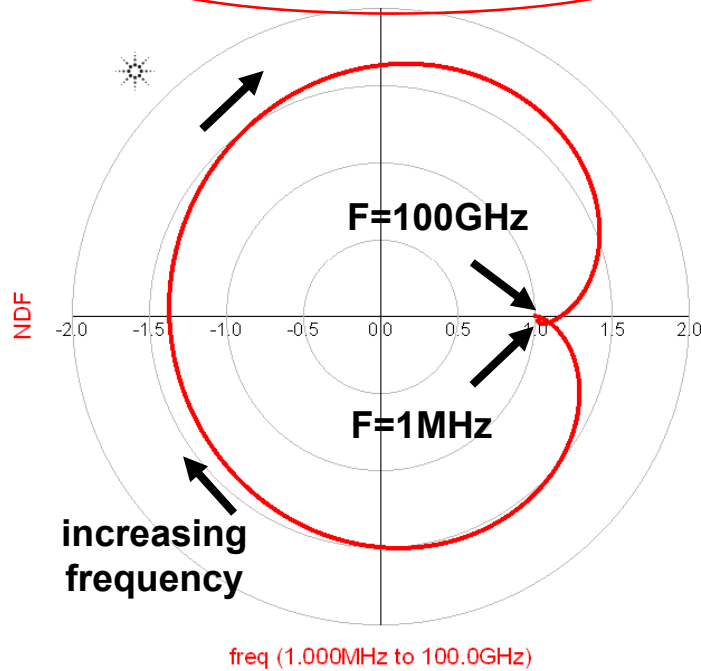
Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Determinants)

NDF is simply the Y matrix determinant at all suspect element nodes divided by the known stable state Y0 matrix determinant

$$\text{Eqn } \text{NDF} = \text{ws_det}(Y[1,:]) / \text{ws_det}(Y[0,:])$$

Number of clockwise encirclements of NDF (frequency from 0 to $+\infty$)

$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$



Clockwise encirclement of the origin shows that the circuit is unstable

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Determinants)

ADS AEL code of ws_det(x) function

```
defun ws_det(X)
{
decl T = X;
decl X_size = size(X);

if (size(X_size) == 1)
{
return (T);
}

if (X_size(1) != X_size(2))
{
error("aelcmd", 2, "ws_det() error", "ws_det() non square matrix");
return;
}

decl N = X_size(1);
decl result = 1;
decl A,B,C,D;

// X = A|B  subdivide the X matrix as follows
//      C|D  where D is a 1x1 matrix
//
// use the equation |X| = |A-BD^-1C|*|D|
// recursively to calculate the determinant of X
// A matrix is upper right portion of X matrix (rows-1 x cols-1)
// B matrix is upper right portion of X matrix (rows-1 x 1)
// C matrix is lower left portion of X matrix (1 x cols-1)
// D matrix is lower right corner of X matrix (1 x 1)

while (N > 1)
{
A = T(1::N-1,1::N-1);
B = T(1::N-1,N);
C = T(N,1::N-1);
D = T(N,N);
T = A-B*C/D;
result = result*D;
--N;
}

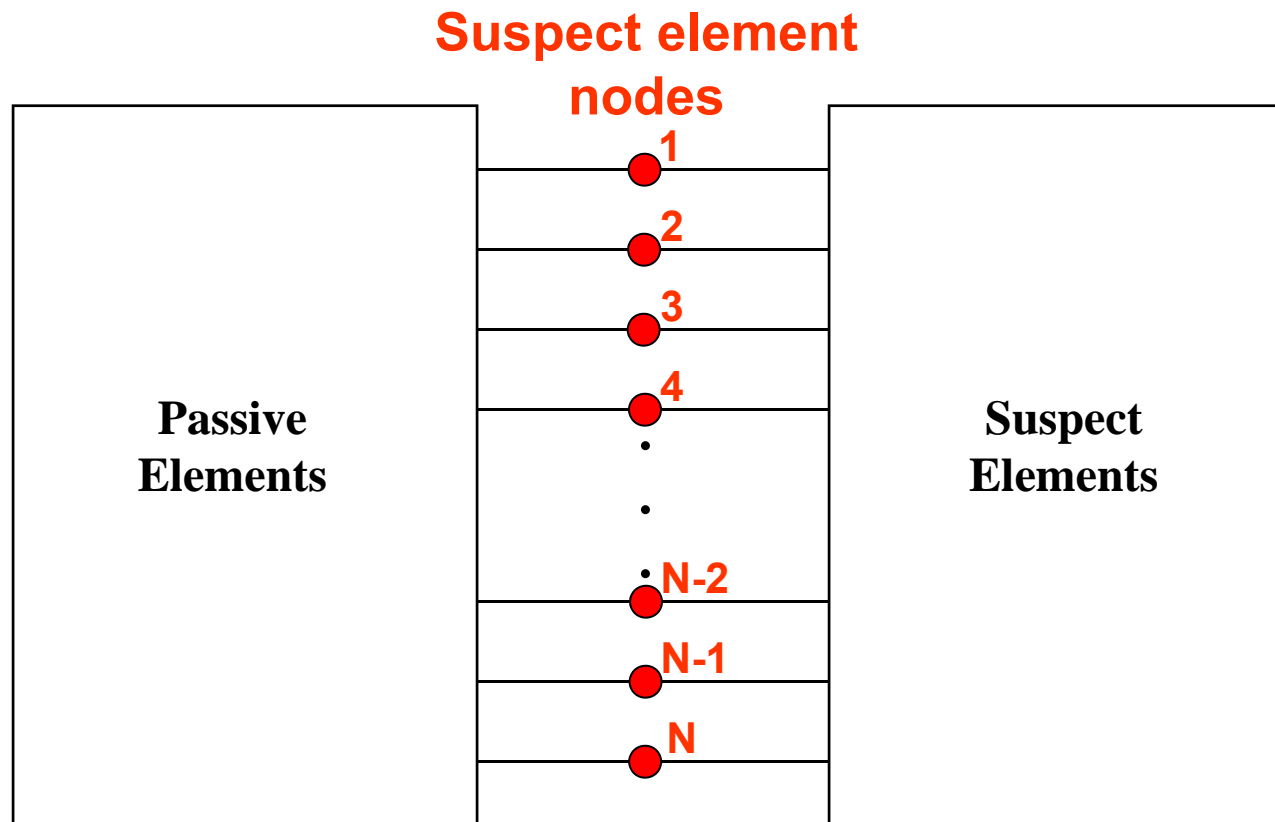
return (result*T);
}
```

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Admittances)

- Next we will show a second closed-loop technique to calculate the NDF function using network admittances.
- We use this technique when we do not have access (in the simulator) to a function that calculates the determinant of the network matrix directly (like the `ws_det()` function I showed for ADS).
- This technique gives exactly the same result as the network determinant approach, it is simply a different way to calculate the network determinants of the NDF function.
- Let's start with our separated network.

NDF from Network Admittances

- If we calculate the admittance looking into node 1 (A_1). The result is equal to: $A_1 = |Y(\omega)|/|Y(\omega)|_{\text{node1 shorted to gnd}}$ (math in appendix 4) where: $Y(\omega)$ is the network admittance matrix at frequency ω .

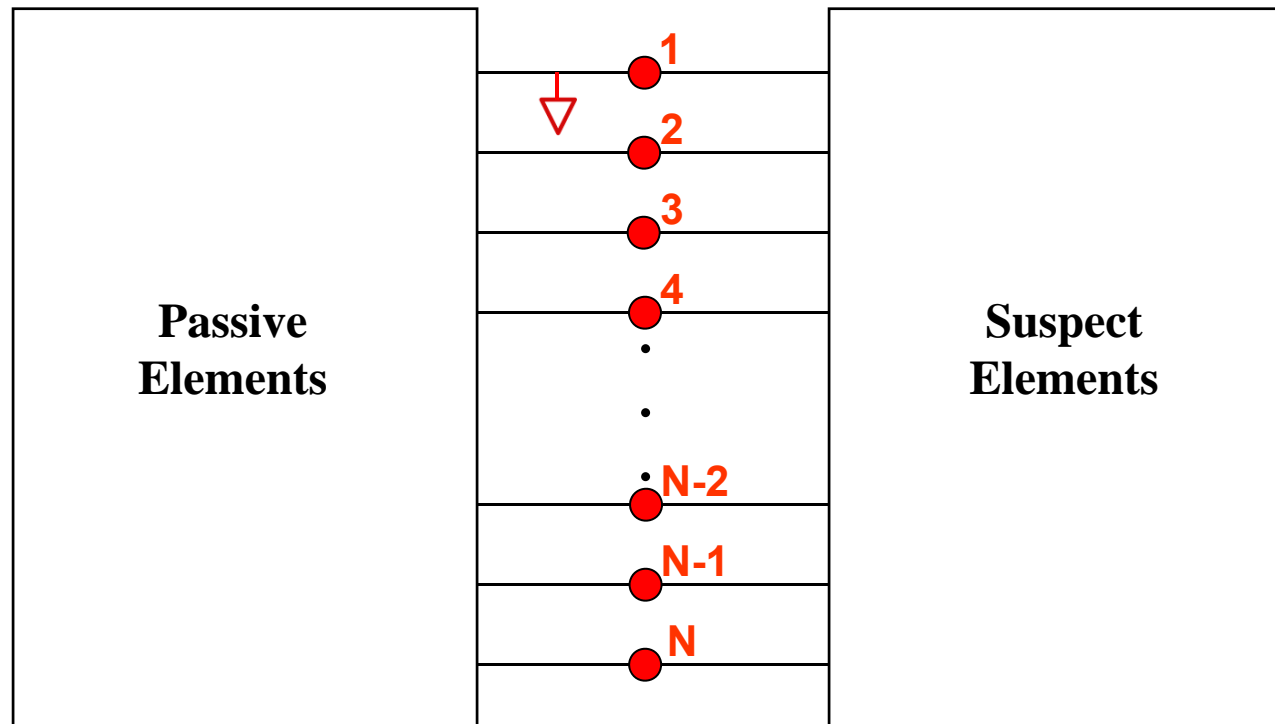


NDF from Network Admittances

- Now if we calculate the admittance looking into node 2 (A_2) with node 1 shorted to ground. The result is equal to:

$$A_2 = |Y(\omega)|_{\text{node1 shorted}} / |Y(\omega)|_{\text{node1 \& node2 shorted}}$$

$$\text{therefore: } A_1 * A_2 = |Y(\omega)| / |Y(\omega)|_{\text{node1 \& node2 shorted}}$$

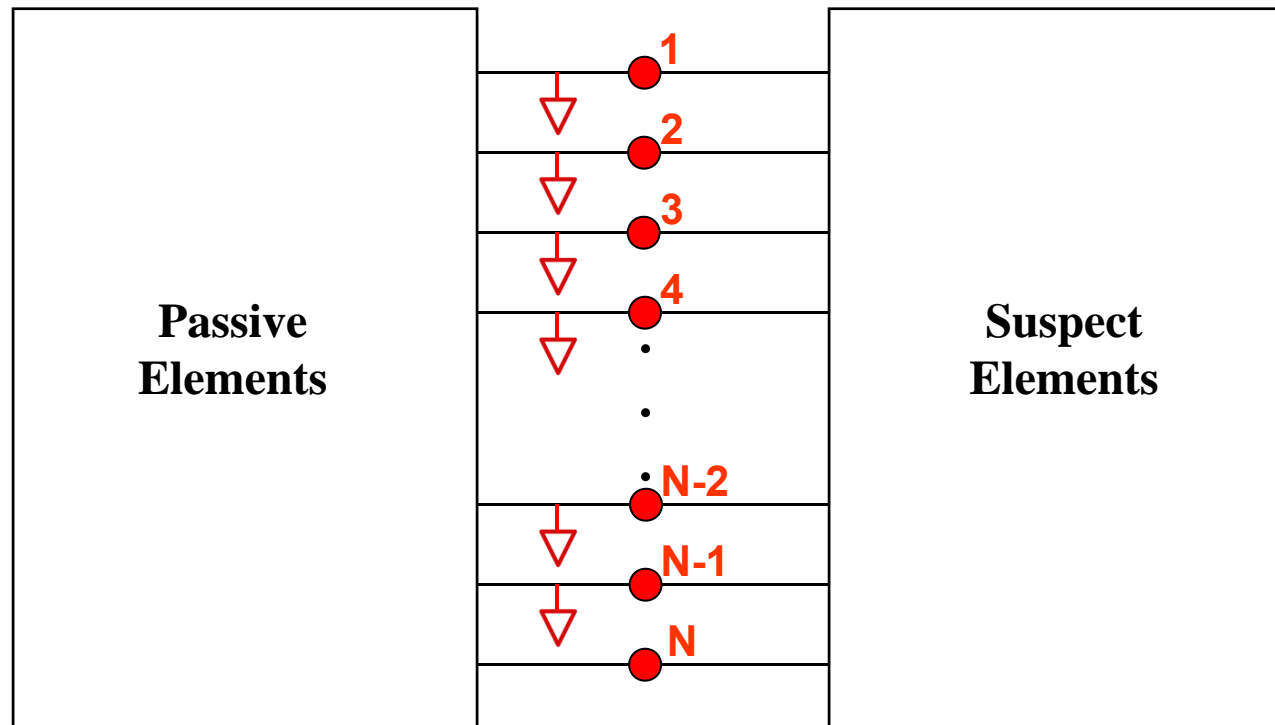


NDF from Network Admittances

- If we continue this process, the admittance looking into node N (A_N) with all prior nodes shorted to ground is equal to:

$$A_N = |Y(\omega)|_{\text{node1 through node N-1 shorted}} / |Y(\omega)|_{\text{node1 through node N shorted}}$$

and therefore: $A_1 * A_2 * \dots * A_N = |Y(\omega)| / |Y(\omega)|_{\text{node1 through node N shorted}}$

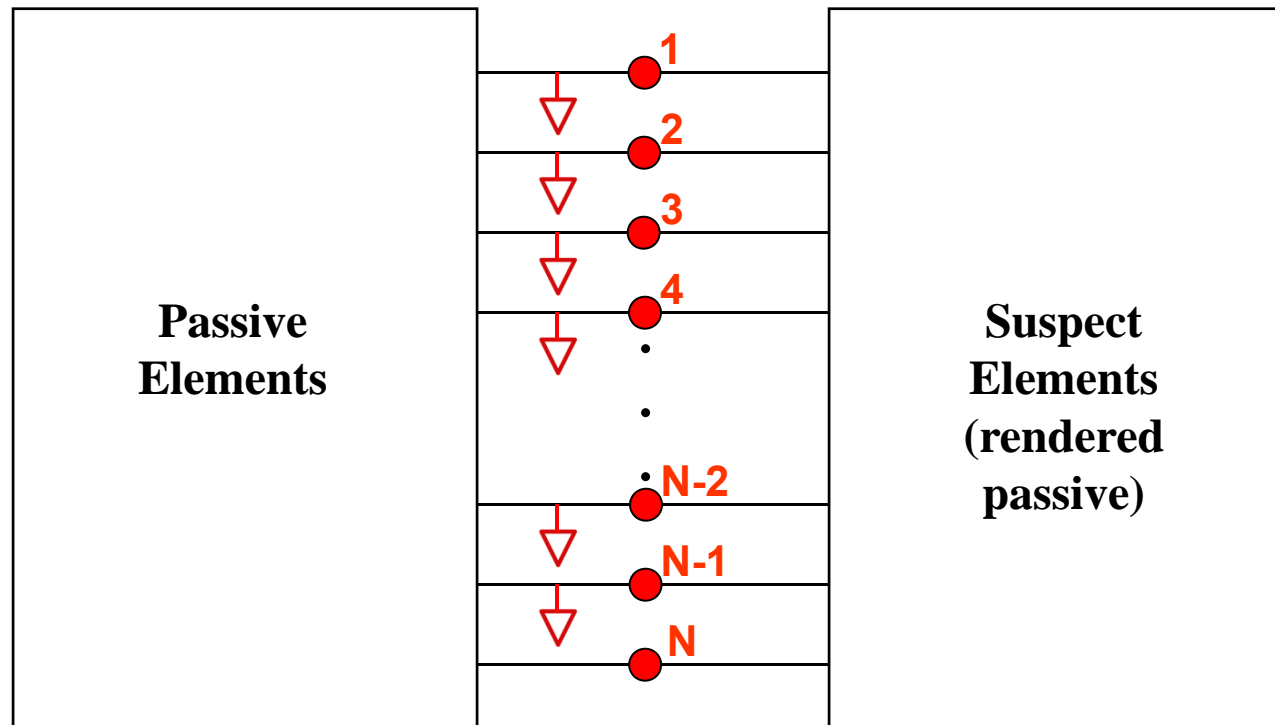


NDF from Network Admittances

- Now we repeat this process with all suspect elements rendered passive (shown with 0 subscripts). The result is equal to:

$$A_{01} * A_{02} * \dots * A_{0N} = |Y_0(\omega)| / |Y_0(\omega)|_{\text{node1 through node N-1 shorted}}$$

and therefore: $(A_1 * A_2 * \dots * A_N) / (A_{01} * A_{02} * \dots * A_{0N}) = |Y(\omega)| / |Y_0(\omega)| = \text{NDF}(\omega)$



Practical Techniques for Stability Analysis (NDF from Network Admittances Summary)

- The procedure to calculate NDF using network admittances is as follows:
 - 1) Calculate the admittance $A_1(\omega)$ looking into the network at a first suspect element node.
 - 2) Calculate the admittance $A_2(\omega)$ looking into the network at second suspect element node (with the first suspect element node shorted).
 - 3) Calculate the admittance $A_3(\omega)$ looking into the network at third suspect element node (with the first and second suspect element nodes shorted).
 - 4) Repeat this procedure up to admittance $A_N(\omega)$ where N is the number of suspect element nodes in the network (with all prior suspect element nodes shorted).
 - 5) Repeat steps 1-4 ($A_{01}(\omega)$ - $A_{0N}(\omega)$) with all of the suspect elements rendered passive.
 - 6) Now,
$$\text{NDF}(\omega) = (A_1 * A_2 * A_3 * \dots * A_N) / (A_{01} * A_{02} * A_{03} * \dots * A_{0N})$$

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator NDF using Network Admittances)

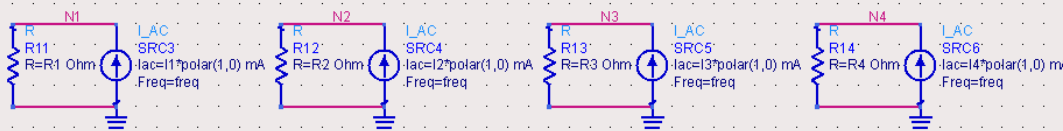
ADS NDF Implementation

Linear Stability Analysis of ring oscillator $NDF = [Y]/[Y0]$

Suspect elements are circled (any dependent sources, and -L, -C, -R's)

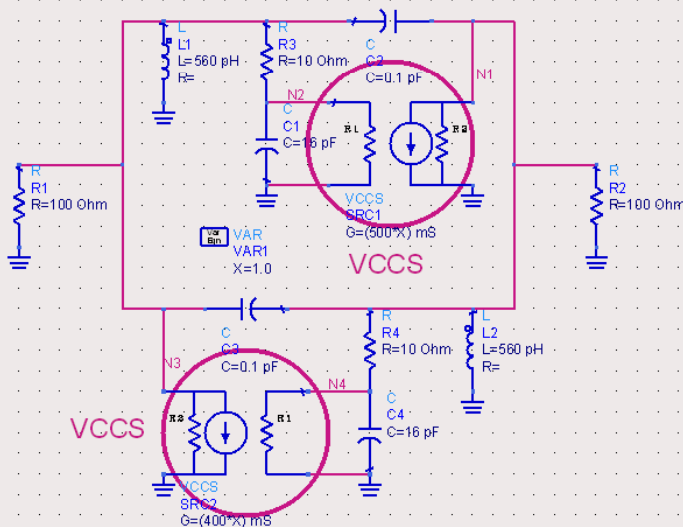
All suspect element nodes must be included in the NDF $[Y]$ and $[Y0]$ matrices:

For each suspect element node admittance we must short all prior nodes to ground:



```

VAR
VAR2
N=1
I1=if (N=1) then 1 else 0 endif
I2=if (N=2) then 1 else 0 endif
I3=if (N=3) then 1 else 0 endif
I4=if (N=4) then 1 else 0 endif
R1=if (N>1) then 1e-9 else 1e9 endif
R2=if (N>2) then 1e-9 else 1e9 endif
R3=if (N>3) then 1e-9 else 1e9 endif
R4=if (N>4) then 1e-9 else 1e9 endif
    
```



parameter sweep variable X
used to toggle between
known stable network
and network to be analyzed
for stability

0-sets dependent sources to zero and -L, -C, -R
values to +L, +C, +R values (known stable network)
1-sets all values to normal (network for stability test)

PARAMETER SWEEP

```

ParamSweep
Sweep1
SweepVar="X"
SimInstanceName[1]="Sweep2"
SimInstanceName[2]=
SimInstanceName[3]=
SimInstanceName[4]=
SimInstanceName[5]=
SimInstanceName[6]=
Start=0
Stop=1
Step=1
    
```

AC

```

AC
AC1
Start=0.001 GHz
Stop=100 GHz
Step=
    
```

N AC frequency sweeps
outputs admittances

PARAMETER SWEEP

```

ParamSweep
Sweep2
SweepVar="N"
SimInstanceName[1]="AC1"
SimInstanceName[2]=
SimInstanceName[3]=
SimInstanceName[4]=
SimInstanceName[5]=
SimInstanceName[6]=
Start=1
Stop=4
Step=1
    
```

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Admittances)

If we don't have access to the Y matrix determinant function NDF can be calculated using network admittances (i.e. voltages) at all suspect element nodes

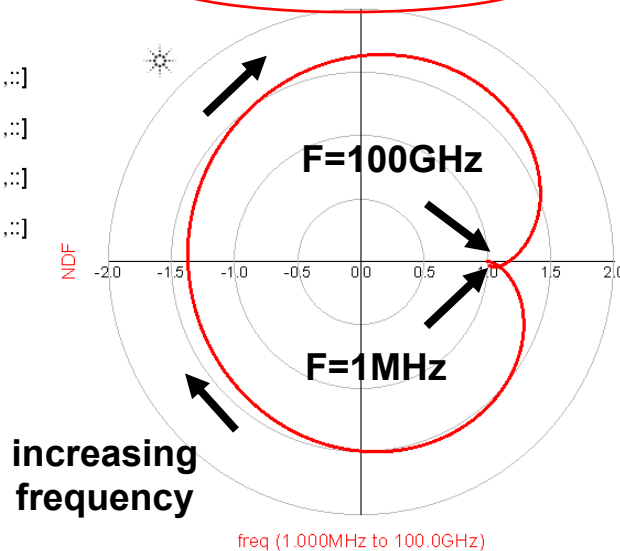
$$\text{Eqn } \text{NDF} = \text{NDF1} * \text{NDF2} * \text{NDF3} * \text{NDF4}$$

$$\text{Eqn } \text{NDF1} = \text{N1}[0,0,:]/\text{N1}[1,0,:]$$

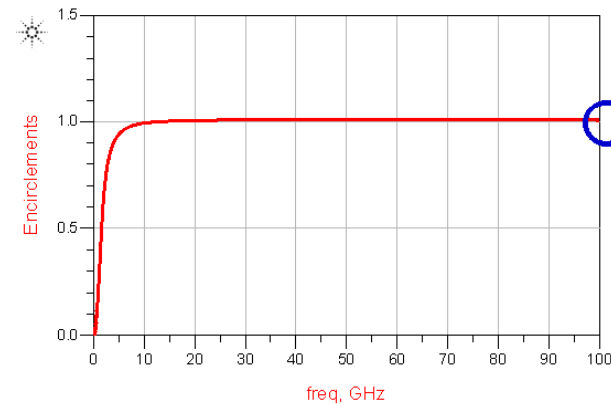
$$\text{Eqn } \text{NDF2} = \text{N2}[0,1,:]/\text{N2}[1,1,:]$$

$$\text{Eqn } \text{NDF3} = \text{N3}[0,2,:]/\text{N3}[1,2,:]$$

$$\text{Eqn } \text{NDF4} = \text{N4}[0,3,:]/\text{N4}[1,3,:]$$



$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$

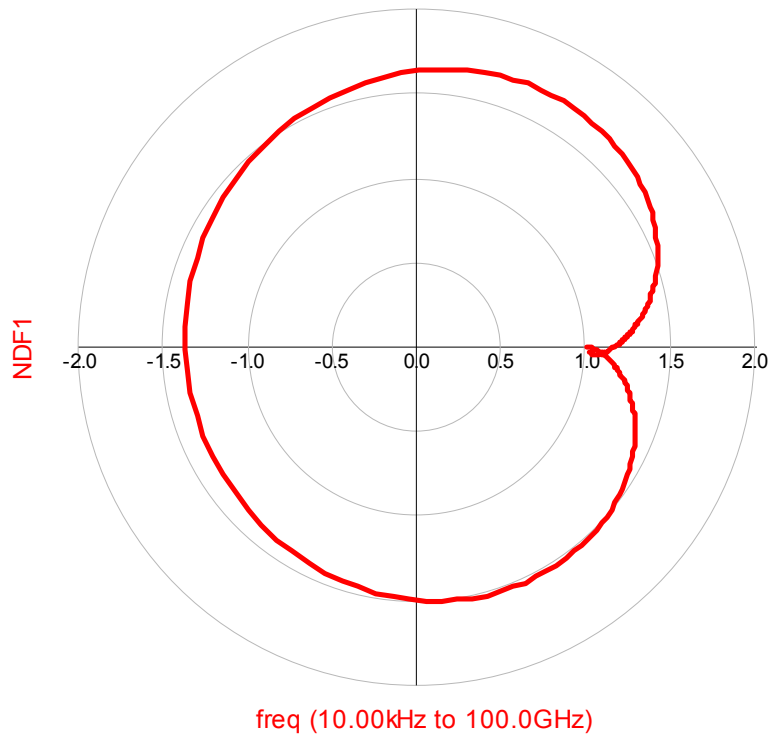


**Number of clockwise encirclements
of NDF (frequency from 0 to $+\infty$)**

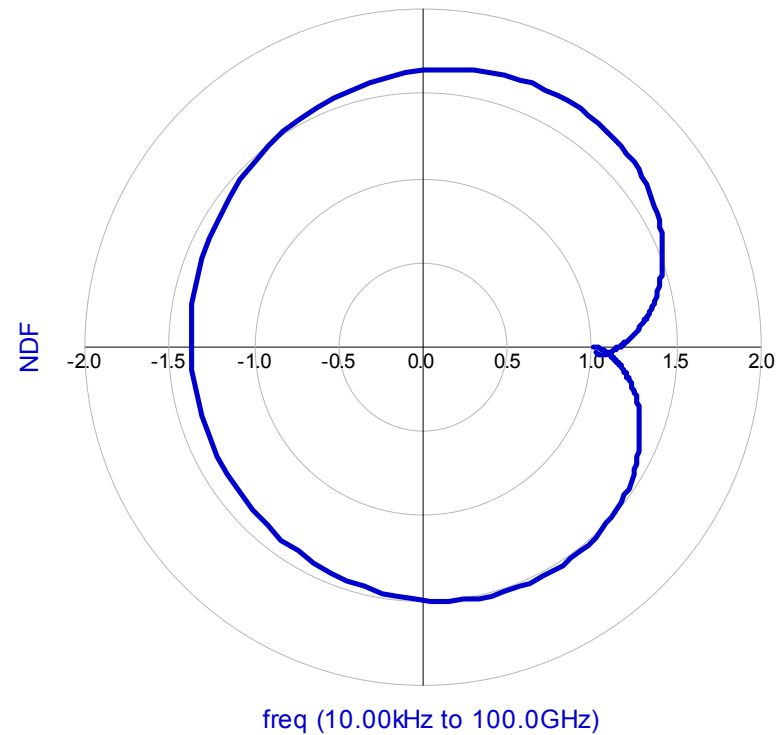
**Clockwise encirclement of the origin
shows that the circuit is unstable**

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator NDF using Network Admittances)

NDF From Network Determinants



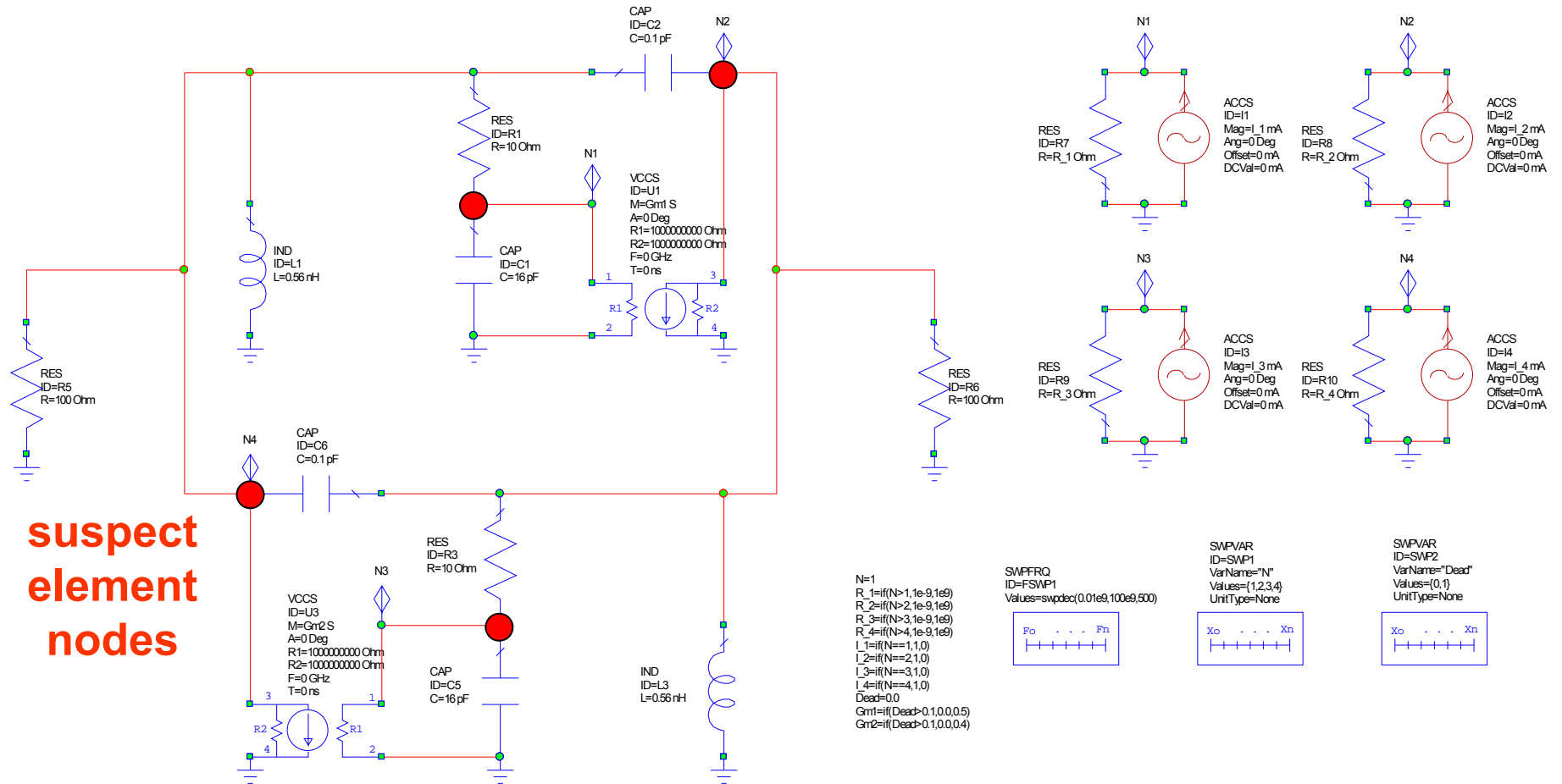
NDF From Network Admittances



Identical results: $NDF(\omega) = |Y(\omega)| / |Y_0(\omega)|$

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator NDF using Network Admittances)

AWR NDF Implementation



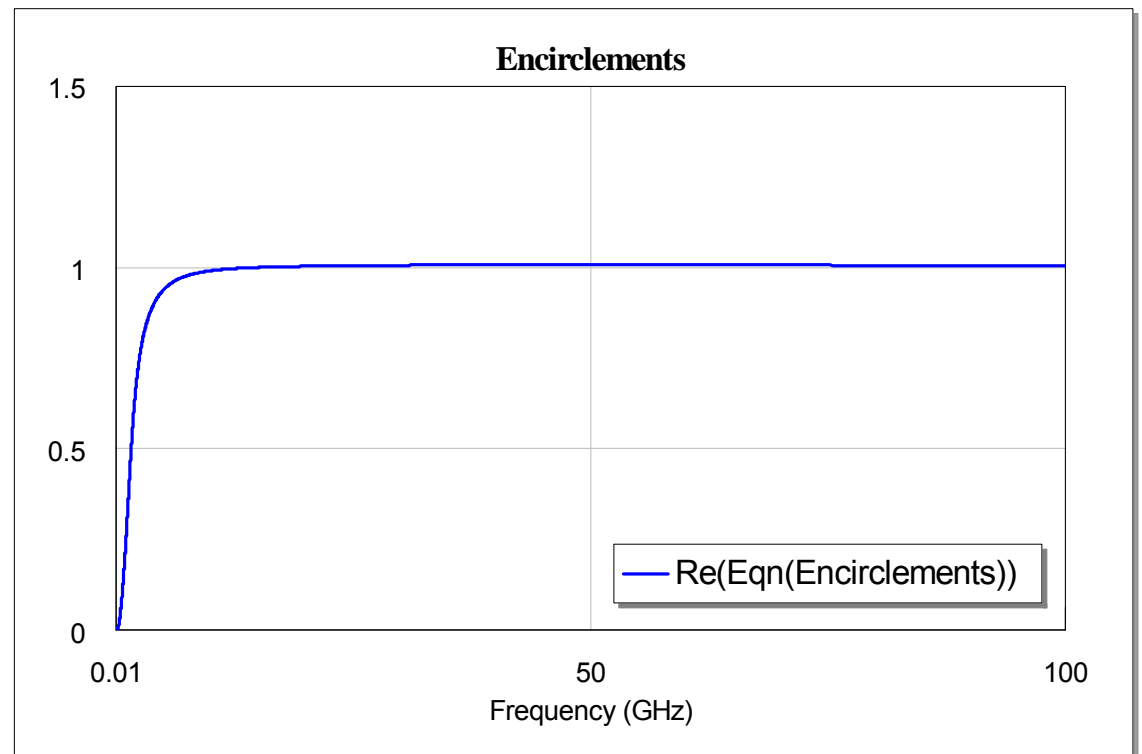
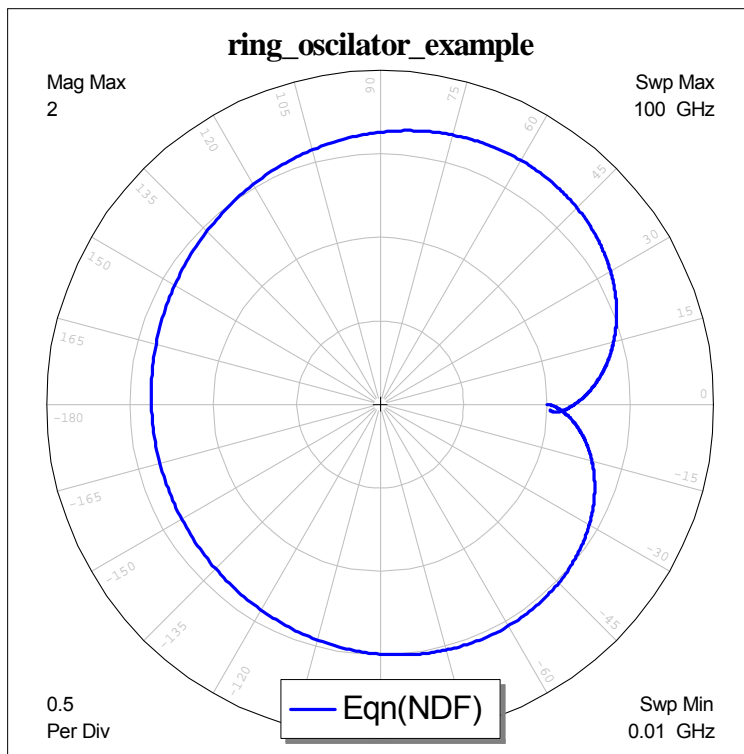
Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Admittances)

AWR NDF Implementation

```
Z1 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I1)[X,1,1]
Z2 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I2)[X,2,1]
Z3 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I3)[X,3,1]
Z4 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I4)[X,4,1]
Z1_0 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I1)[X,1,2]
Z2_0 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I2)[X,2,2]
Z3_0 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I3)[X,3,2]
Z4_0 = ring_oscillator_example.AP.$FSWP1:Vac(ACCS.I4)[X,4,2]
NDF=(Z1_0*Z2_0*Z3_0*Z4_0)/(Z1*Z2*Z3*Z4)
NDF_phase = Output Equations:AngU(Eqn(NDF))
Encirclements=NDF_phase/(-2*_PI)
```

Practical Techniques for Linear Stability Analysis (Example1 ring oscillator: NDF using Network Admittances)

AWR NDF Implementation



Practical Techniques for Stability Analysis

- OK, so it doesn't matter if you use Network Determinants directly or Network Admittances to calculate the NDF (we get the same answer).
- Let's look at some another examples using non-foster components (i.e. negative L's & C's).
- We will use Network Determinants to calculate the NDF.
- The first example uses the original ring oscillator example where we replaced the 10Ω resistors with their non-foster equivalents.
- This should introduce 2 more encirclements in the NDF plot.
- Then we will reduce the Gm's of the VCCS elements and show that we get one less encirclement (one less frequency of oscillation) but still maintain two from the non-foster elements.
- Next, we will replace one of the non-foster circuits back with the original 10Ω resistor and see that we now only have one encirclement remaining.
- Finally, we will show that the NDF used at any single node can be used to "probe" the network and find out which suspect elements (and feedback loops) within the circuit are the source of the instabilities.
- This also works for non-linear networks (more on those later)...

Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

Linear Stability Analysis of networks including non-Foster elements $NDF = [Y]/[Y_0]$
 Suspect elements are circled (any dependent sources, and -L, -C, -R's)
 All suspect element nodes must be included in the NDF [Y] and [Y0] matrices

Replaced 10Ω resistors with non-foster -L -C equivalents

suspect element nodes

N1 Term1 Num=1 Z=50 Ohm
 N2 Term2 Num=2 Z=50 Ohm
 N3 Term3 Num=3 Z=50 Ohm
 N4 Term4 Num=4 Z=50 Ohm
 N5 Term5 Num=5 Z=50 Ohm
 N6 Term6 Num=6 Z=50 Ohm
 N7 Term7 Num=7 Z=50 Ohm
 N8 Term8 Num=8 Z=50 Ohm

S-PARAMETERS

S_Param
 SP1
 Start=0.001 GHz
 Stop=1.000 GHz
 Step=
 Dec=200
 CalcY=yes

PARAMETER SWEEP

ParamSweep
 Sweep1
 SweepVar="X"
 SimInstanceName[1]="SP1"
 SimInstanceName[2]=
 SimInstanceName[3]=
 SimInstanceName[4]=
 SimInstanceName[5]=
 SimInstanceName[6]=
 Start=0
 Stop=1
 Step=1

S-parameter frequency sweep outputs Y-parameter matrix

parameter sweep variable X used to toggle between known stable network and network to be analyzed for stability

0-sets dependent sources to zero and -L, -C, -R values to +L, +C, +R values (known stable network)
 1-sets all values to normal (network for stability test)

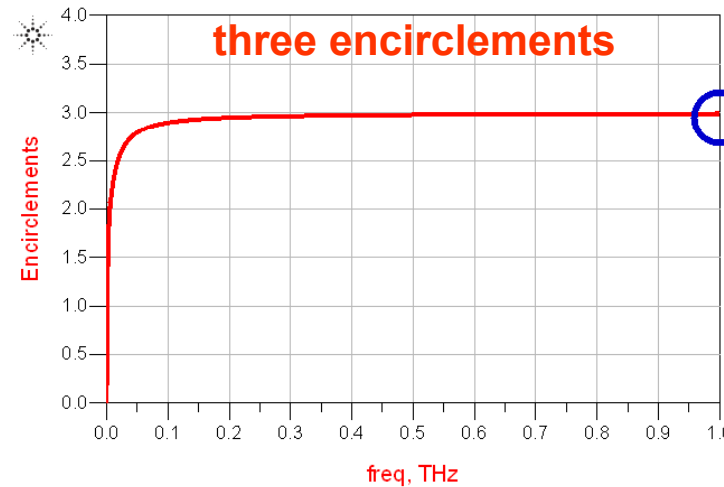
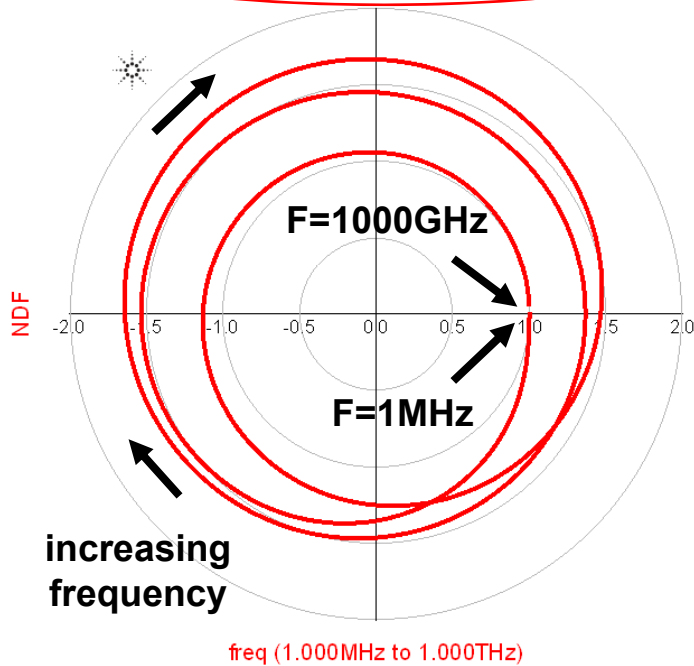
Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

NDF is simply the Y matrix determinant at all suspect element nodes divided by the known stable state Y0 matrix determinant

$$\text{Eqn } \text{NDF} = \text{ws_det}(Y[1,:]) / \text{ws_det}(Y[0,:])$$

Number of clockwise encirclements of NDF (frequency from 0 to $+\infty$)

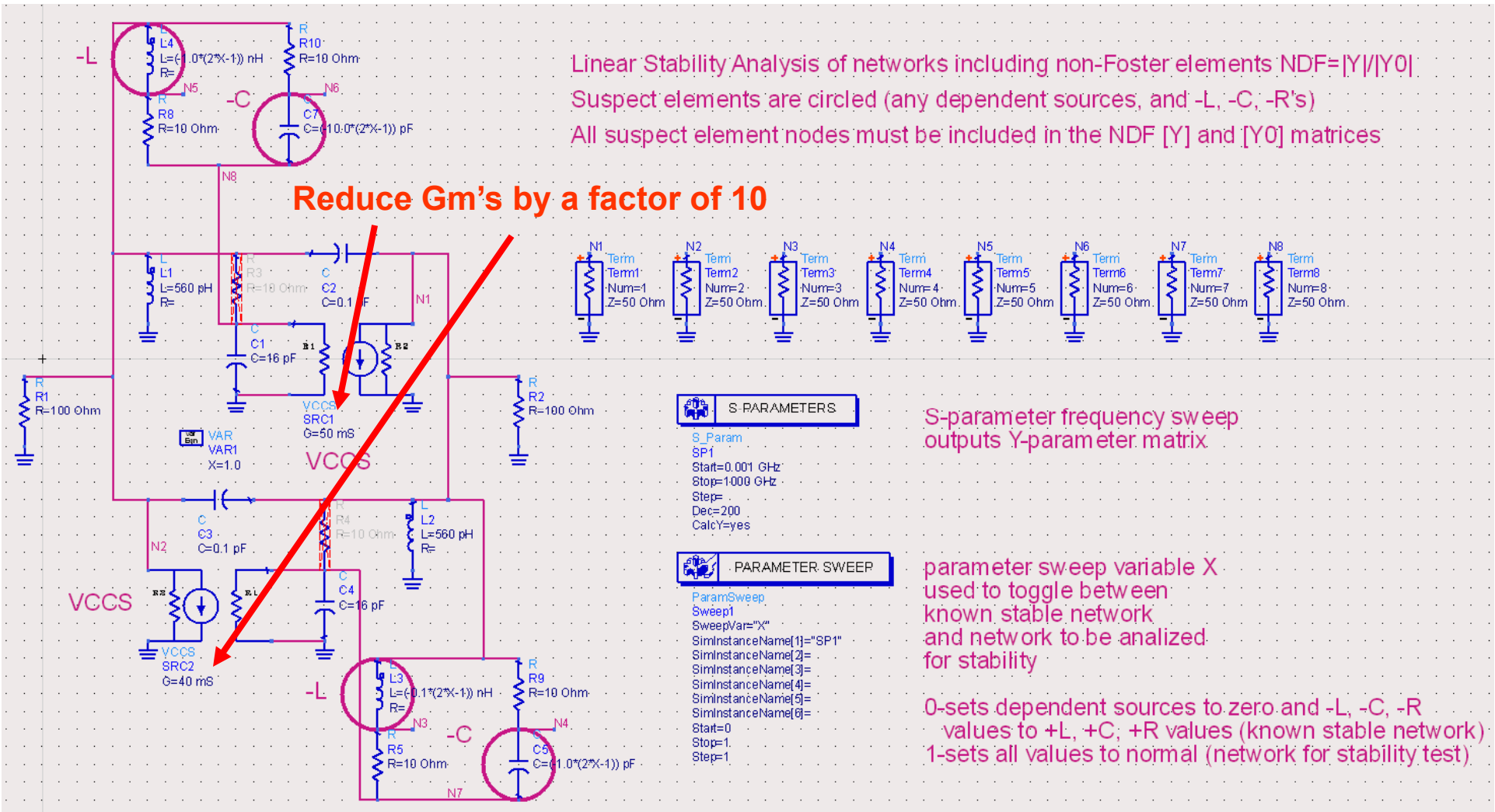
$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$



Three RHP (right half plane) zeroes detected. Circuit is UNSTABLE!

Clockwise encirclement of the origin shows that the circuit is unstable

Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)



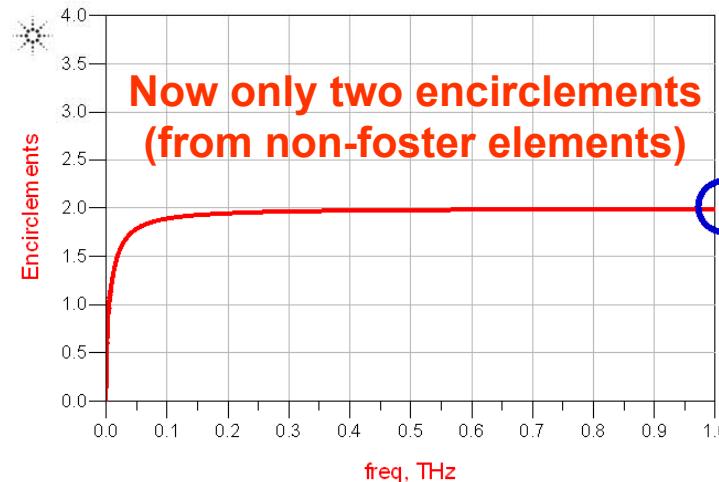
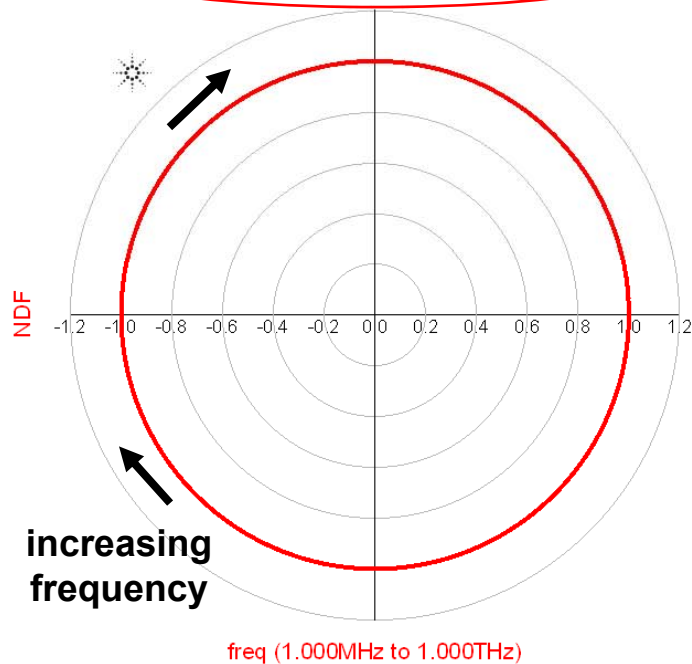
Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

NDF is simply the Y matrix determinant at all suspect element nodes divided by the known stable state Y0 matrix determinant

$$\text{Eqn } \text{NDF} = \text{ws_det}(Y[1, :]) / \text{ws_det}(Y[0, :])$$

Number of clockwise encirclements of NDF (frequency from 0 to $+\infty$)

$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$



Clockwise encirclement of the origin shows that the circuit is unstable

Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

Linear Stability Analysis of networks including non-Foster elements NDF= $|Y|/|Y_0|$
 Suspect elements are circled (any dependent sources, and -L, -C, -R's)
 All suspect element nodes must be included in the NDF [Y] and [Y0] matrices

Reduce Gm's by a factor of 10

Back to 10Ω resistor

N1	Term1	Num=1	Z=50 Ohm
N2	Term2	Num=2	Z=50 Ohm
N3	Term3	Num=3	Z=50 Ohm
N4	Term4	Num=4	Z=50 Ohm
N5	Term5	Num=5	Z=50 Ohm
N6	Term6	Num=6	Z=50 Ohm
N7	Term7	Num=7	Z=50 Ohm
N8	Term8	Num=8	Z=50 Ohm

S-PARAMETERS

S_Param
 SP1
 Start=0.001 GHz
 Stop=1000 GHz
 Step=
 Dec=200
 CalcY=yes

PARAMETER SWEEP

ParamSweep
 Sweep1
 SweepVar="X"
 SimInstanceName[1]="SP1"
 SimInstanceName[2]=
 SimInstanceName[3]=
 SimInstanceName[4]=
 SimInstanceName[5]=
 SimInstanceName[6]=
 Start=0
 Stop=1
 Step=1

S-parameter frequency sweep outputs Y-parameter matrix

parameter sweep variable X used to toggle between known stable network and network to be analyzed for stability

0-sets dependent sources to zero and -L, -C, -R values to +L, +C, +R values (known stable network)
 1-sets all values to normal (network for stability test)

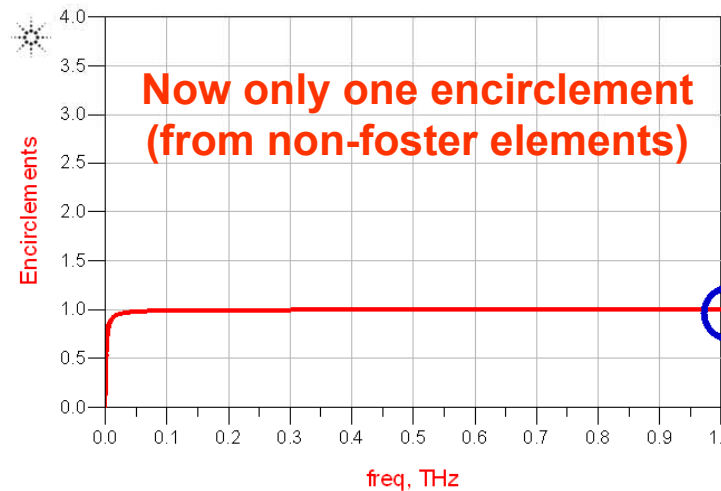
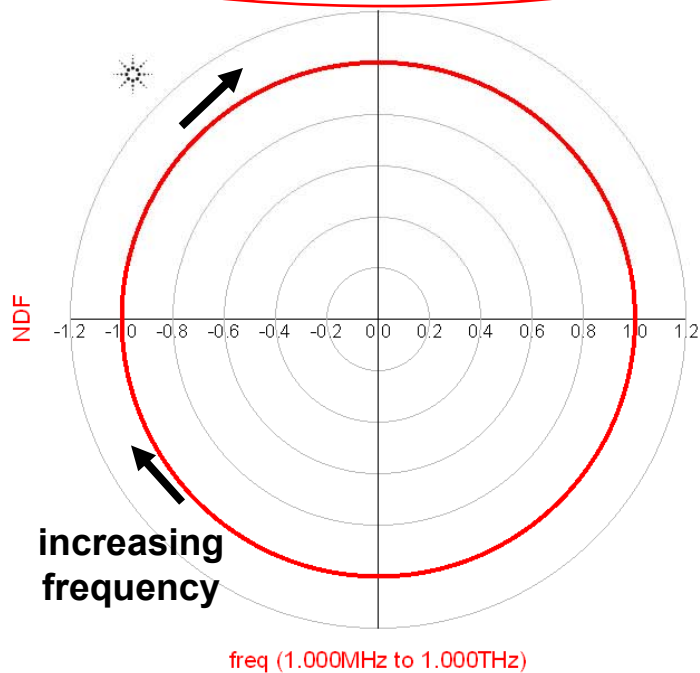
Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

NDF is simply the Y matrix determinant at all suspect element nodes divided by the known stable state Y0 matrix determinant

$$\text{Eqn } \text{NDF} = \frac{\text{ws_det}(Y[1,:])}{\text{ws_det}(Y[0,:])}$$

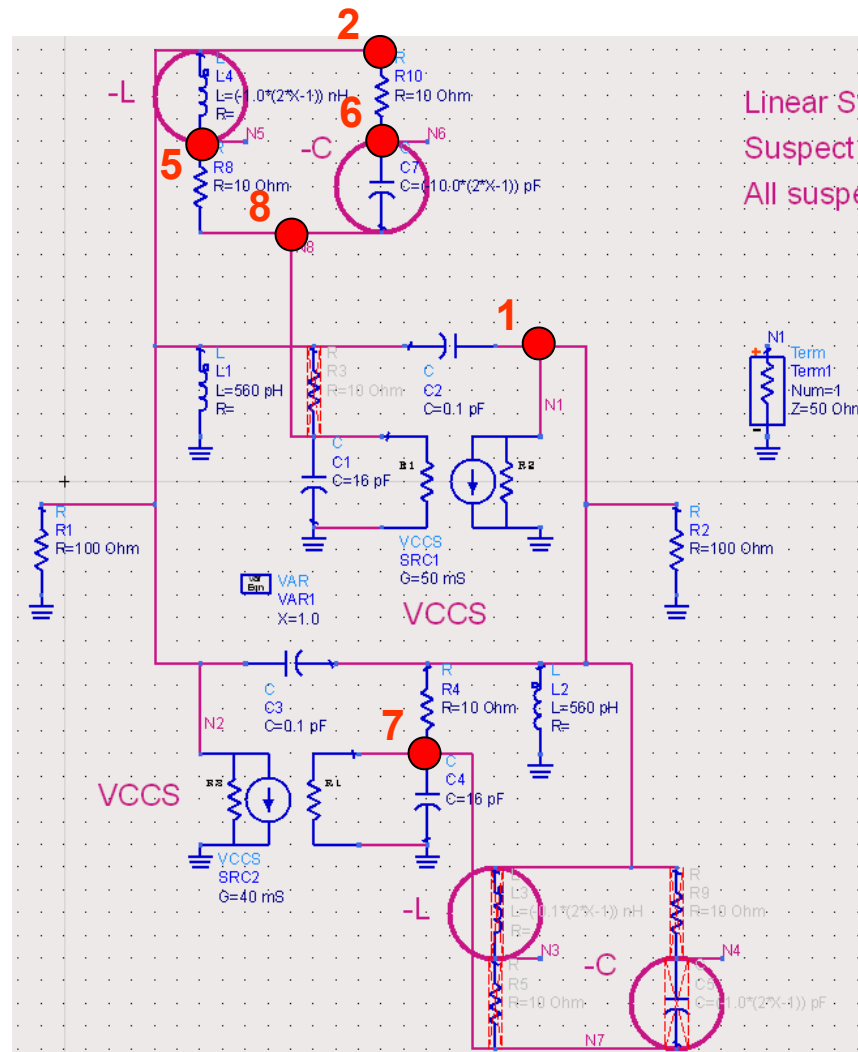
Number of clockwise encirclements of NDF (frequency from 0 to $+\infty$)

$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$



Clockwise encirclement of the origin shows that the circuit is unstable

Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

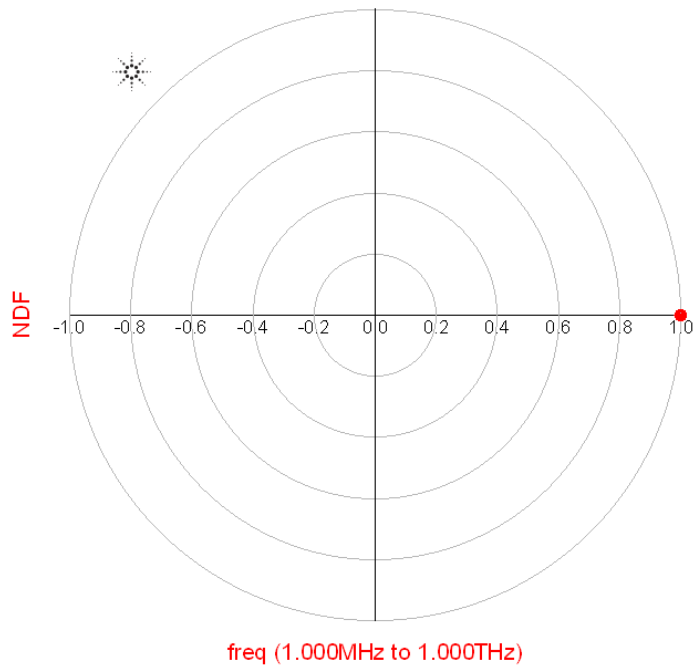


Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

**Results probing at Node 7 only
(circuit is unstable, yet no encirclement?)**

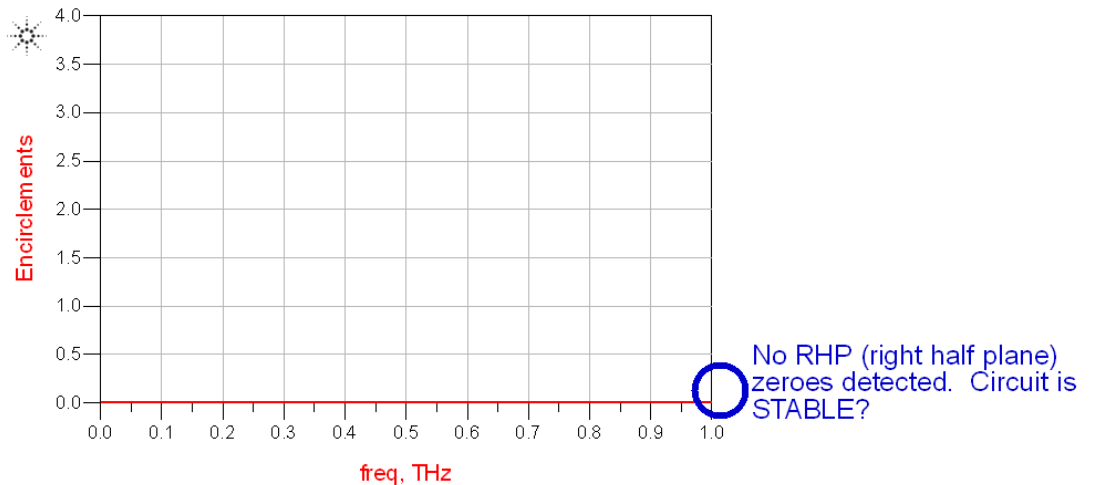
NDF is simply the Y matrix determinant at all suspect element nodes divided by the known stable state Y0 matrix determinant

$$\text{Eqn } \text{NDF} = \text{ws_det}(Y[1,::]) / \text{ws_det}(Y[0,::])$$



Node 7 is not in the unstable circuit loop

$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$

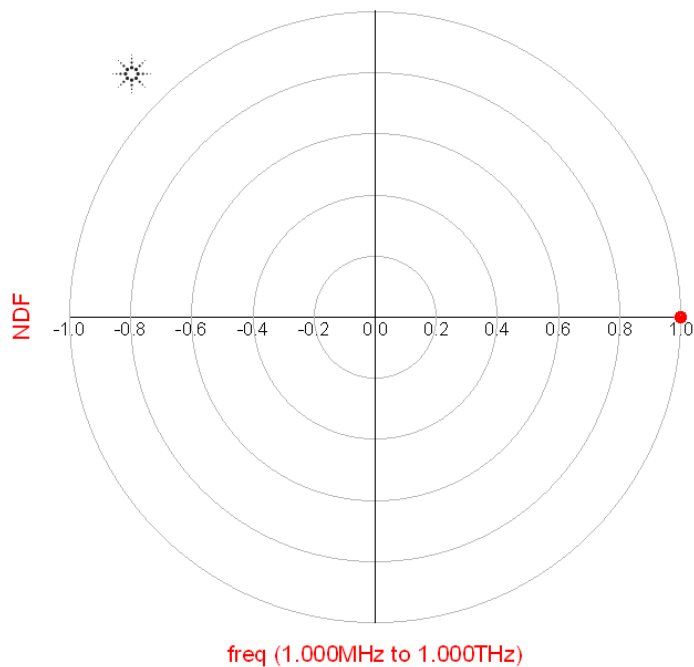


Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

Results probing at Node 1 only
(circuit is unstable, yet no encirclement?)

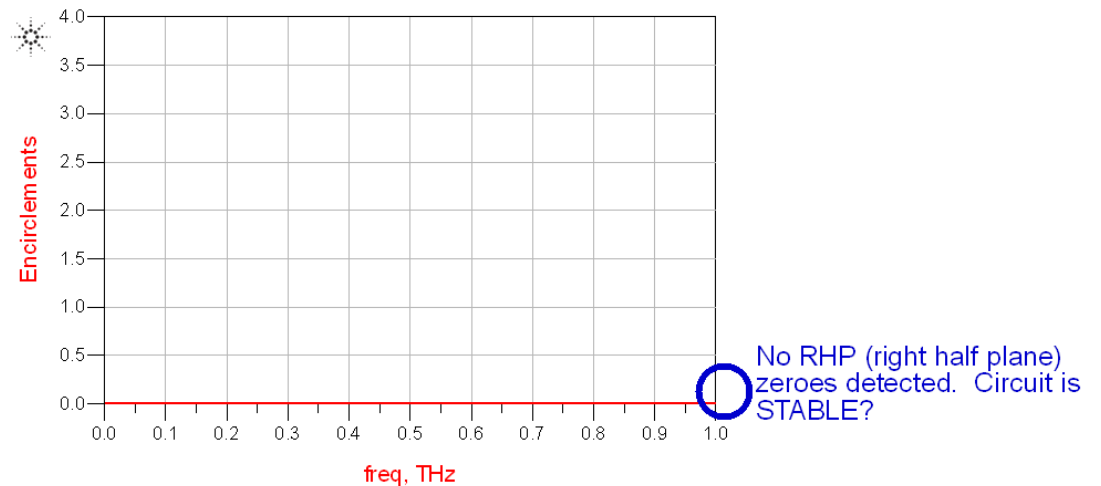
NDF is simply the Y matrix determinant at all suspect element nodes
divided by the known stable state Y0 matrix determinant

$$\text{Eqn } \text{NDF} = \text{ws_det}(Y[1,::]) / \text{ws_det}(Y[0,::])$$



Node 1 is also not in the unstable circuit loop

$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$

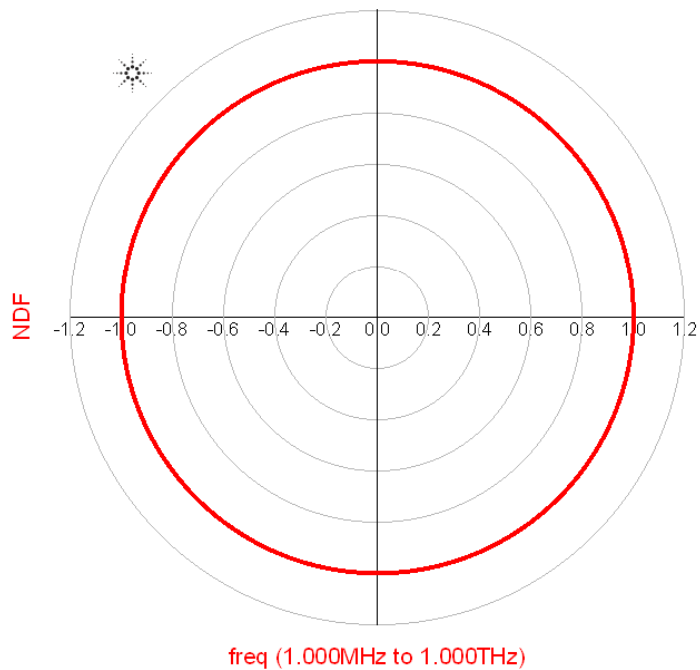


Practical Techniques for Linear Stability Analysis (Example2 NDF of ring oscillator with non-foster components)

**Results probing at Nodes 2,5,6 & 8 only
(circuit is unstable, and shows encirclement)**

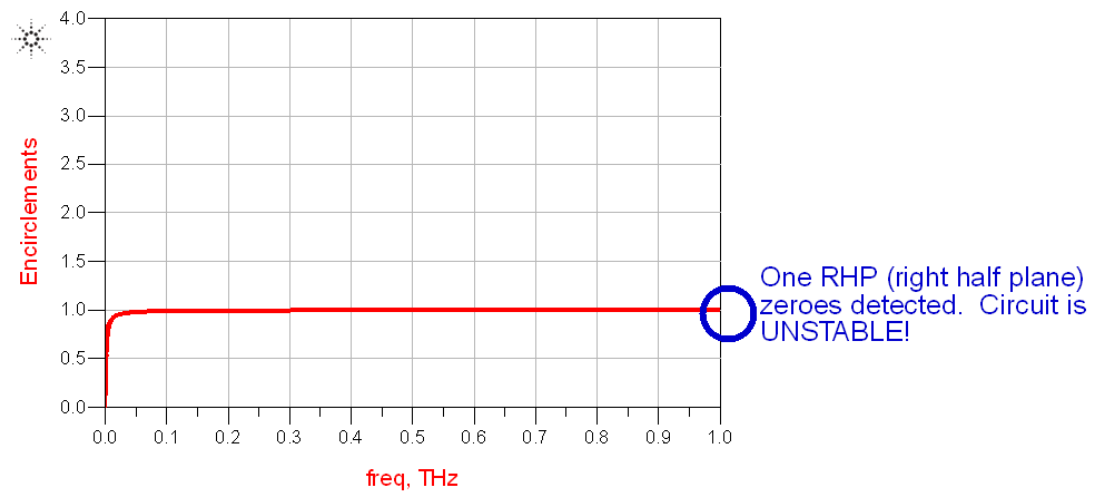
NDF is simply the Y matrix determinant at all suspect element nodes divided by the known stable state Y0 matrix determinant

$$\text{Eqn } \text{NDF} = \text{ws_det}(Y[1,:]) / \text{ws_det}(Y[0,:])$$



Nodes 2,5,6 & 8 are in the unstable circuit loop

$$\text{Eqn } \text{Encirclements} = (\text{unwrap}(\text{phase}(\text{NDF})) - \text{unwrap}(\text{phase}(\text{NDF}[0]))) / -360$$



Practical Techniques for Stability Analysis

- OK, now let's get back to the question on how do we render transistors (and [S]-parameter files of transistors) passive in order to calculate $|Y_0(\omega)|$ in $NDF = |Y(\omega)| / |Y_0(\omega)|$?
- In other words, how do we get a “passive” device model if we only have “black box” transistor models (or [S]-parameter files)?
- Is there a way to find RHP zeroes of the network determinant using “black box” active device models only? The answer is YES!
- Remember, we can use any determinant $|Y(s)|_{\text{stable}}$ to normalize the network determinant $|Y(s)|$

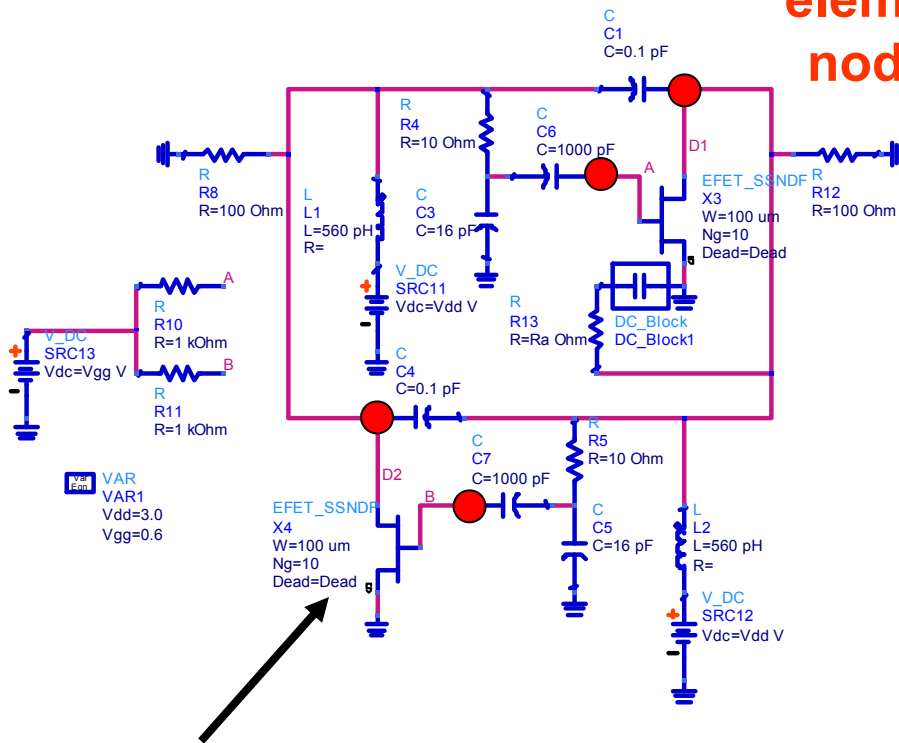
$$NDF = |Y(s)| / |Y(s)|_{\text{stable}}$$

where $|Y(s)|_{\text{stable}}$ is the determinant of the same (or same rank) network from a known stable state (i.e. contains no RHP zeroes).

- This is the general case of the NDF where $|Y(s)|_{\text{stable}} = |Y_0(s)|$ is a specific case.
- For example, we can use a known stable bias point...
(gate bias below pinchoff, or zero drain bias as examples)
- We can also use “dead” device models to render transistors (or [S]-parameter files of transistors) passive.
- Let's consider the following examples...

Practical Techniques for Linear Stability Analysis (Example3 ring oscillator NDF using Network Admittances)

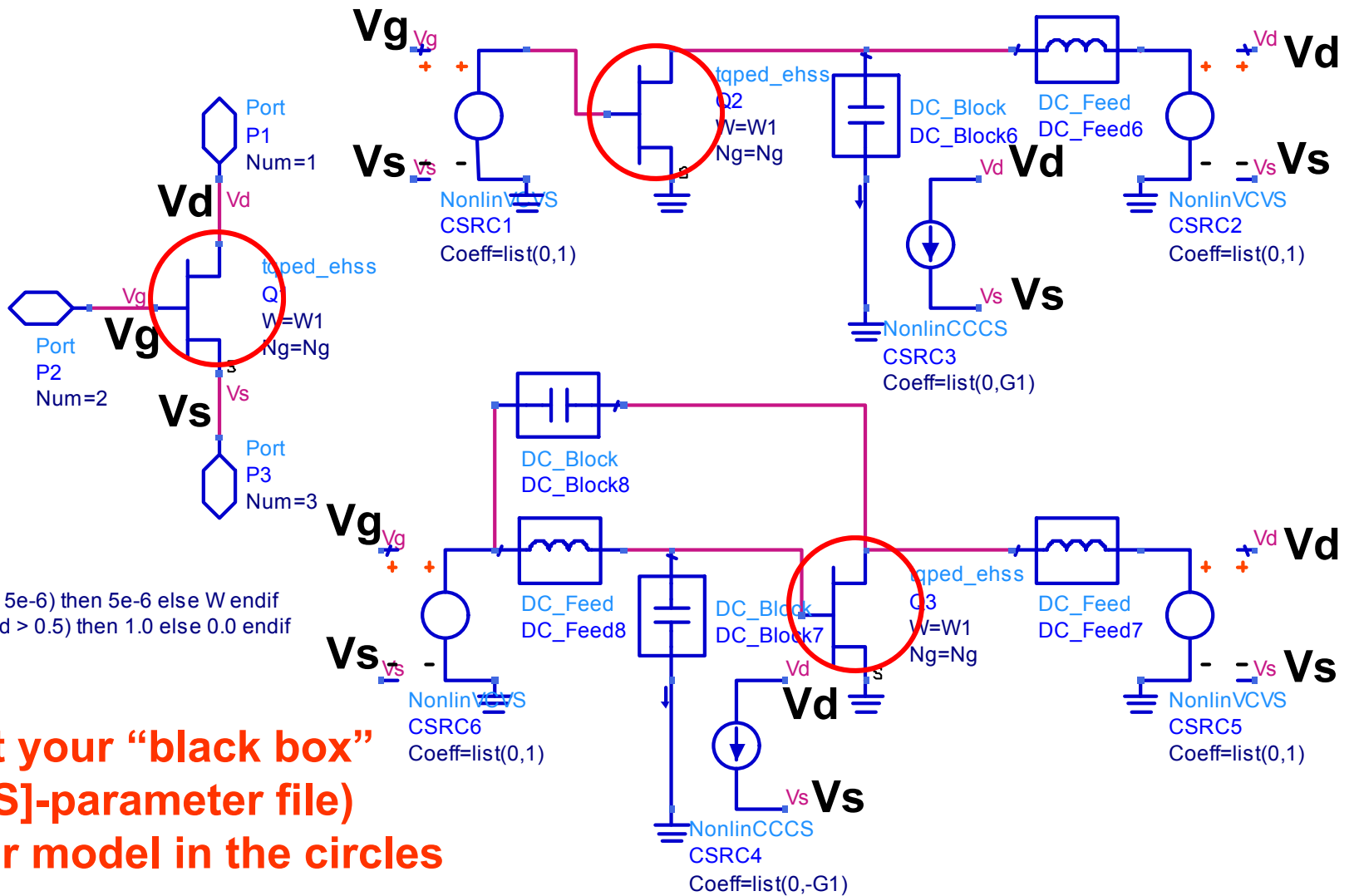
suspect
element
nodes



Requires “dead device” model
and 2N frequency sweeps

Case 1: “dead” device from a
“black box” transistor model
to normalize $|Y(s)|_{\text{stable}} = |Y(s)|_{(gm=0)}$

Dead Device “black box” Model, Dead = 0 for Active Mode Dead = 1 same as gm = 0 (see Appendix 2)

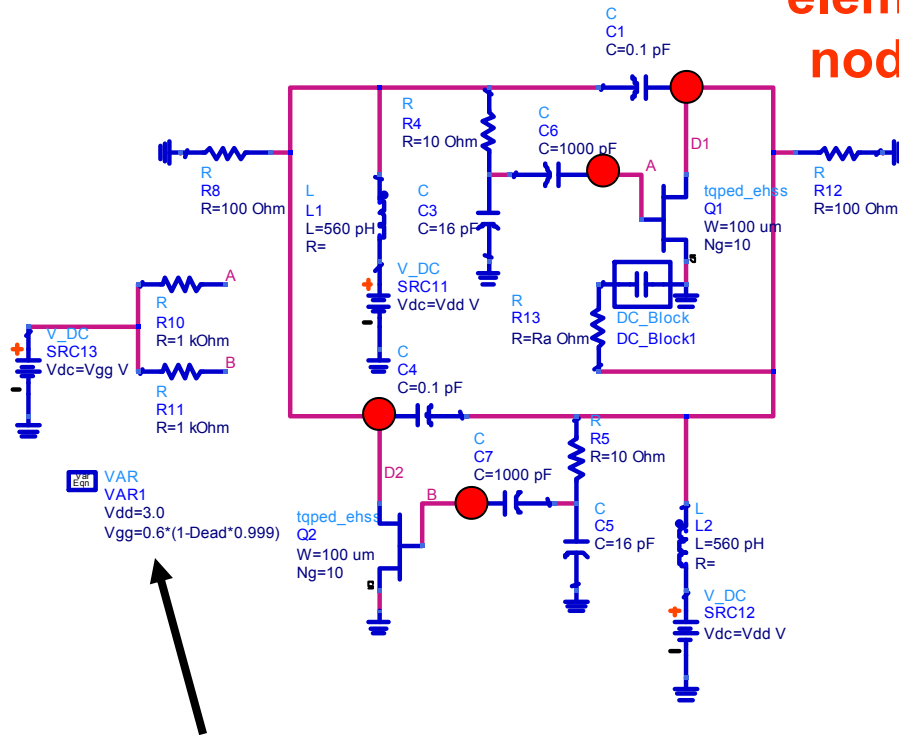


VAR
Eqn
 VAR1
 W1=if (W < 5e-6) then 5e-6 else W endif
 G1=if (Dead > 0.5) then 1.0 else 0.0 endif

Insert your “black box”
 (or [S]-parameter file)
 transistor model in the circles

Practical Techniques for Linear Stability Analysis (Example3 ring oscillator NDF using Network Admittances)

suspect
element
nodes



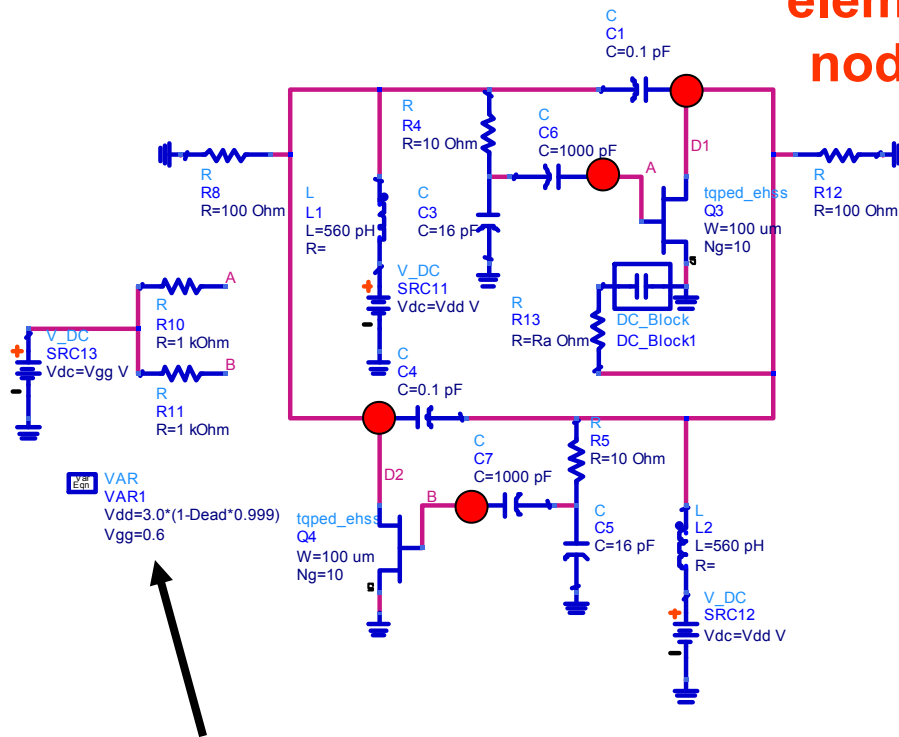
No “special” models required
2N frequency sweeps

Case 2: V_{gg} below pinchoff
(known stable state)

to normalize $|Y(s)|_{\text{stable}} = |Y(s)|_{(V_{gg}=0)}$

Practical Techniques for Linear Stability Analysis (Example3 ring oscillator NDF using Network Admittances)

suspect
element
nodes

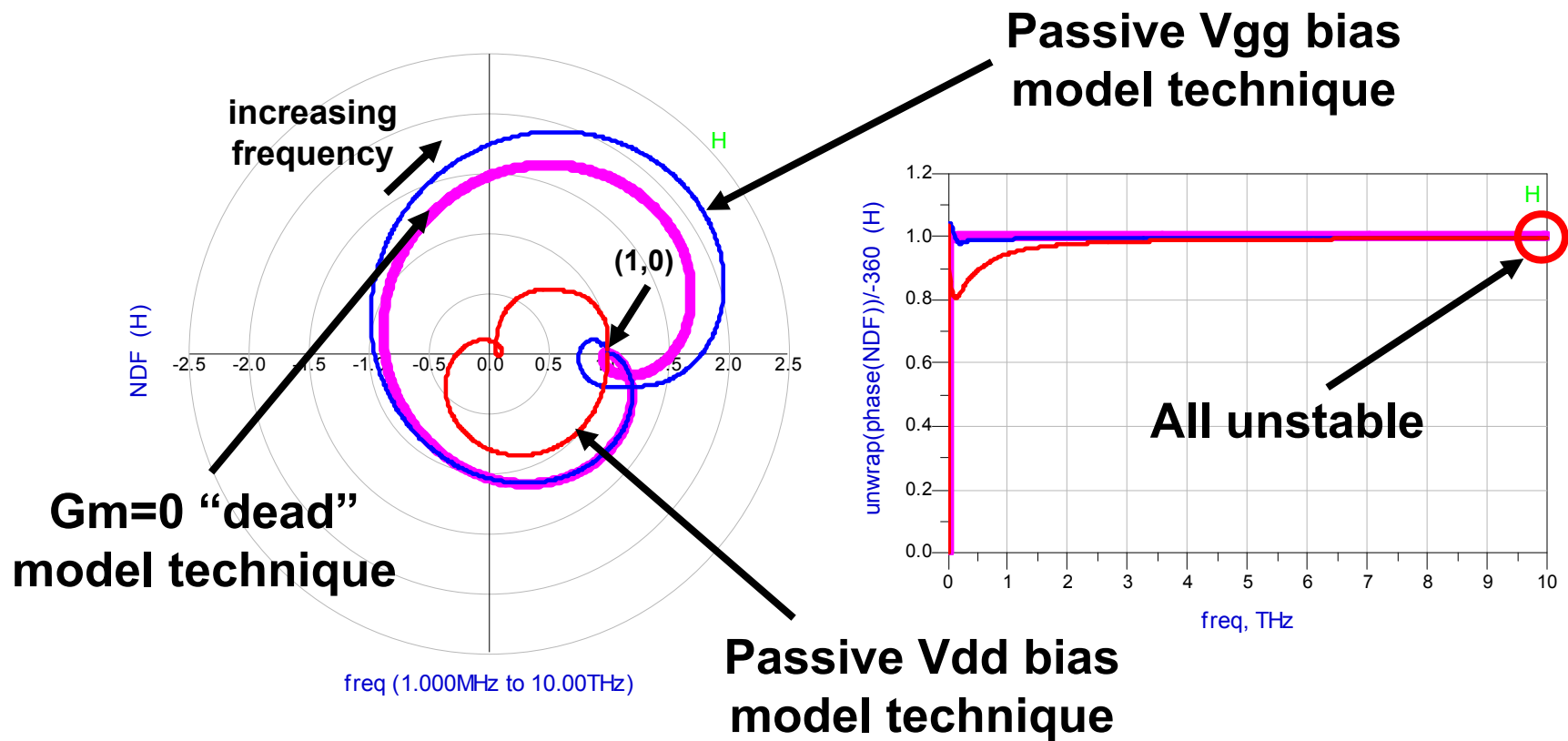


No “special” models required
2N frequency sweeps

**Case 3: Drain Bias $V_{dd} = 0$
(known stable state)**

to normalize $|Y(s)|_{\text{stable}} = |Y(s)|_{(V_{dd}=0)}$

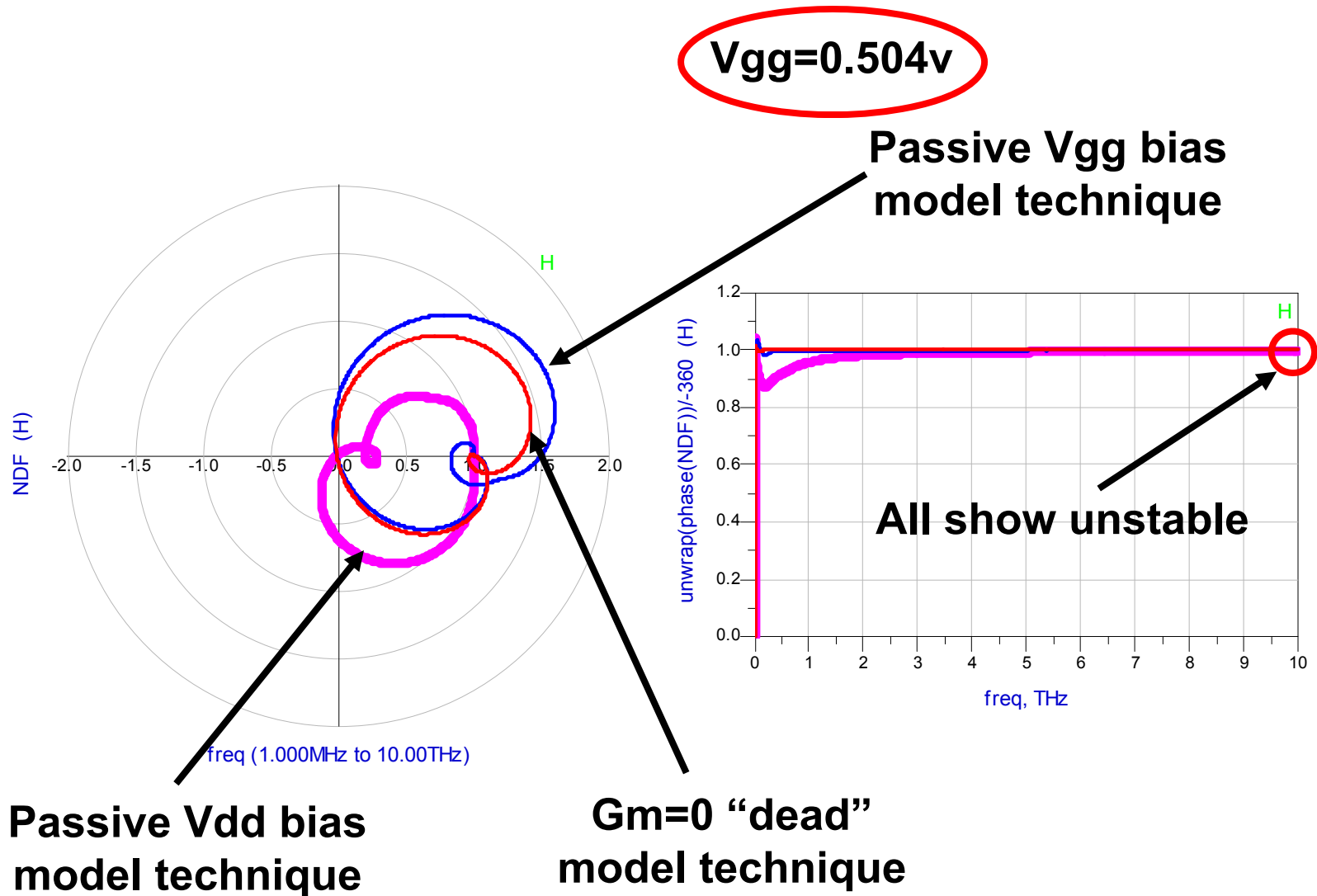
Practical Techniques for Linear Stability Analysis (Example3 ring oscillator NDF using Network Admittances)



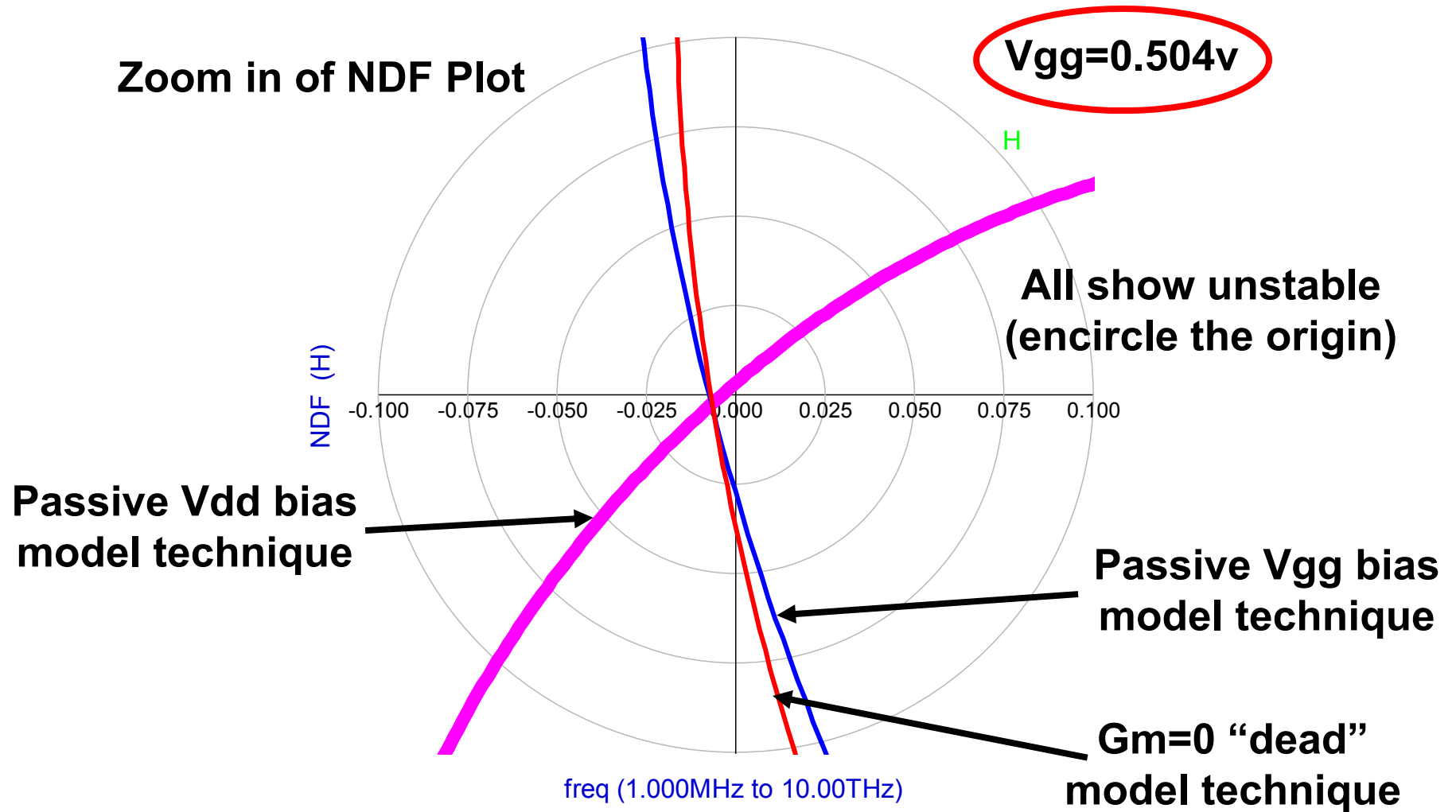
Practical Techniques for Stability Analysis

- The three plots look different, but all encircle the origin.
- The shape of the NDF plot is unimportant, what matters is if there are clockwise encirclements of the origin.
- So what happens if the circuit is barely stable/unstable?
- Are we still able to detect stability correctly?
- The answer is yes. It doesn't matter how you normalize the NDF (as long as the rank of $|Y(s)|_{\text{stable}}$ is the same as $|Y(s)|$ and $|Y(s)|_{\text{stable}}$ contains no RHP zeroes, i.e. is stable).
- We can tell that $|Y(s)|_{\text{stable}}$ is of the same rank as $|Y(s)|$ by observing that NDF approaches $(1,0)$ at $\omega = +\infty$.
- The following examples versus gate bias demonstrate this fact...
 - $V_{gg}=0.504\text{v}$ (barely unstable case)
 - $V_{gg}=0.505\text{v}$ (barely stable case)

Practical Techniques for Linear Stability Analysis (Example3 ring oscillator NDF using Network Admittances)



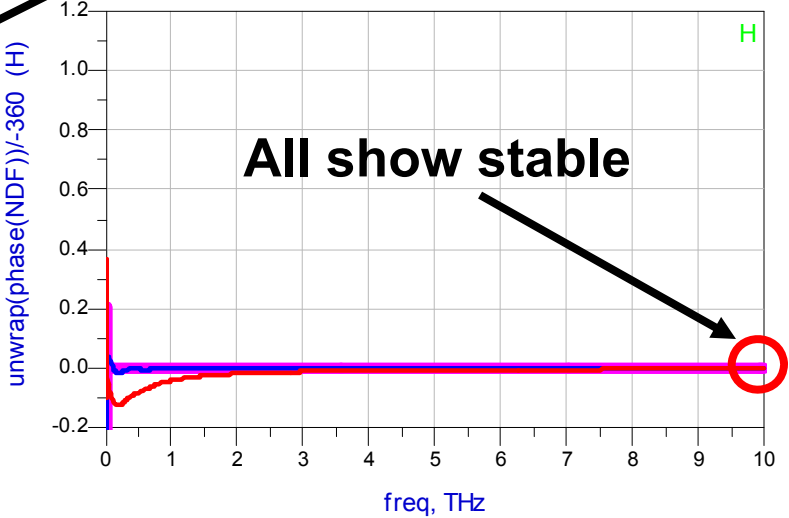
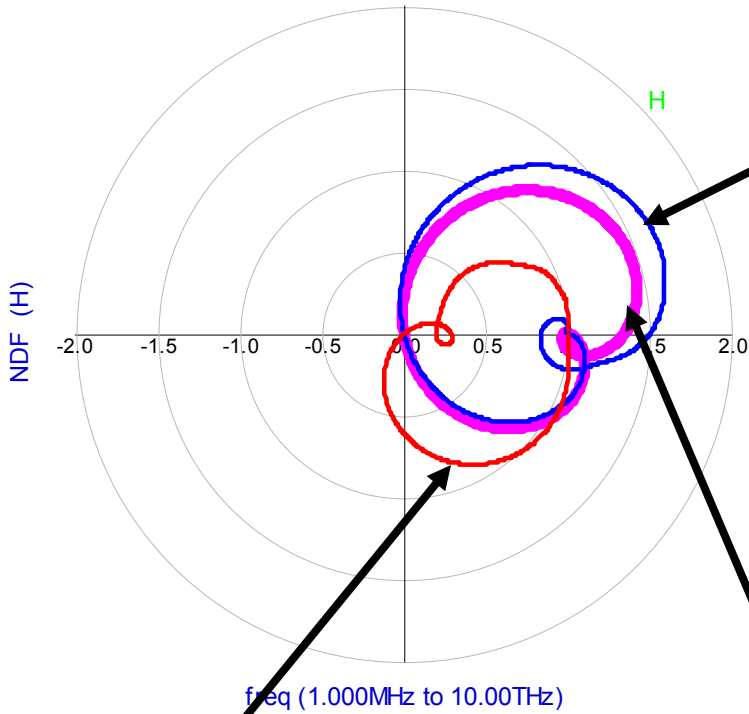
Practical Techniques for Linear Stability Analysis (Example3 ring oscillator NDF using Network Admittances)



Practical Techniques for Linear Stability Analysis (Example3 ring oscillator NDF using Network Admittances)

V_{gg}=0.505v

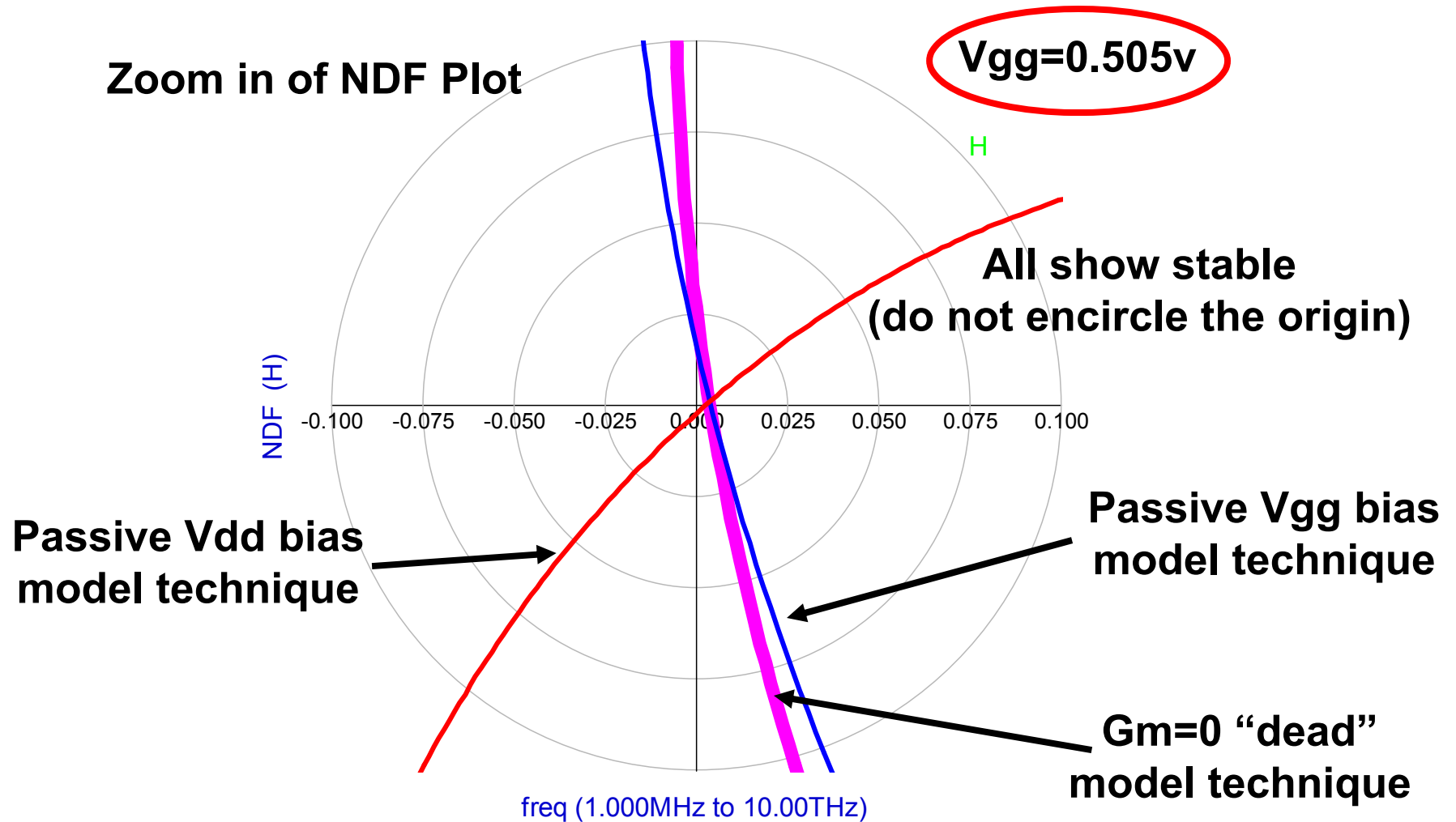
**Passive V_{gg} bias
model technique**



**Passive V_{dd} bias
model technique**

**G_m=0 “dead”
model technique**

Practical Techniques for Linear Stability Analysis (Example 3 ring oscillator NDF using Network Admittances)





Rigorous Nonlinear Network Stability Theory

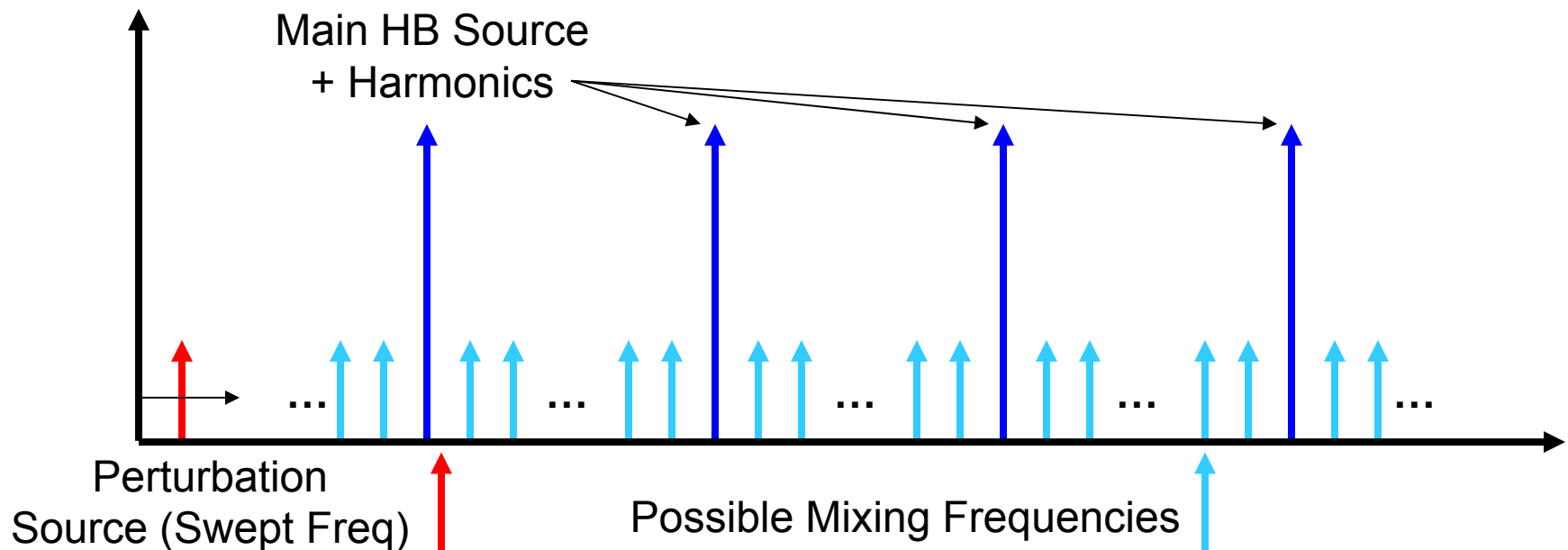
Non-linear NDF Stability Analysis

- The NDF stability analysis techniques used in linear networks are easily extended to non-linear networks using a Harmonic Balance swept perturbation technique (in place of [S]-parameter or AC sweeps).
- Just as the NDF of a linear network is used to determine the stability of a particular “DC” operating condition, the NDF of a nonlinear network is used to determine the stability of a particular nonlinear “steady-state” operating condition (i.e. input power, frequency, etc.).
- The nonlinear NDF determinant calculations are performed by introducing a small perturbing current (or voltage) source to calculate admittances at each suspect element node (just as for linear networks).
- **The perturbation frequency must be non-harmonically related to any driving source frequencies** and swept from $\omega = 0$ to $+\infty$ just as in the linear case to capture all stability information.
- In the nonlinear case (using the admittance technique), the prior suspect nodes are shorted to ground but only at the perturbing frequency.
- Also, for the nonlinear case, we must include dependent charge sources (as well as dependent current and voltage sources) as suspect elements. We will show an example to demonstrate this fact...

NDF of Nonlinear Ring Oscillator Example

HB Frequency Map for Nonlinear NDF Analysis

- By introducing a small perturbing source (to a correct HB “steady-state” solution), and allowing for possible mixing frequencies, we can use NDF to detect if that solution is stable (i.e. whether other frequencies “oscillations” need to be included to achieve a correct “steady-state” solution).
- This means that the HB solution including the perturbation must lie within the local neighborhood of the nonlinear operating point of the network. (i.e. **the perturbing source cannot be too large in magnitude**)



Non-linear NDF Stability Analysis (NDF from Network Admittances Summary)

- The procedure to calculate nonlinear NDF using network admittances is as follows:
 - 1) Calculate the admittance at the perturbing frequency $A_1(\omega)$ looking into the network at a first suspect element node.
 - 2) Calculate the admittance at the perturbing frequency $A_2(\omega)$ looking into the network at second suspect element node (with the first suspect element node shorted at the perturbing frequency).
 - 3) Calculate the admittance at the perturbing frequency $A_3(\omega)$ looking into the network at third suspect element node (with the first and second suspect element nodes shorted at the perturbing frequency).
 - 4) Repeat this procedure up to admittance $A_N(\omega)$ where N is the number of suspect element nodes in the network (with all prior suspect element nodes shorted at the perturbing frequency).
 - 5) Repeat steps 1-4 ($A_{01}(\omega)$ - $A_{0N}(\omega)$) with all of the suspect elements rendered passive.
 - 6) Now,
$$\text{NDF}(\omega) = (A_1 * A_2 * A_3 * \dots * A_N) / (A_{01} * A_{02} * A_{03} * \dots * A_{0N})$$

Non-linear NDF Stability Analysis

- The following example will demonstrate this technique using a non-linear ring oscillator (with a single nonlinear VCCS element).
- NDF is calculated using the admittance technique using two separate two-tone analyses (there is only one nonlinear element in this circuit).
- The first tone (F1 – 1GHz) drives the ring oscillator circuit into the non-linear operating regime.
- The second tone is a small perturbation current at F2 that is swept from 10MHz to 100GHz using a log frequency sweep.
- $P_{in} = -50\text{dBm}$ at 1GHz is used for the normalization (i.e. known stable state) of the NDF in this analysis.
- Network determinant calculations at this power level can be used for normalization because we have verified that the network is stable with zero input power using the NDF technique for linear networks first (NDF for the linear case has two dependent source calculations).
- Using low power for normalization also allows for accurate oscillation frequency identification. (as we will demonstrate)
- This ring oscillator example was chosen to demonstrate the detection of parametric oscillations (oscillations that happen at large-signal operation over some finite driven power range).

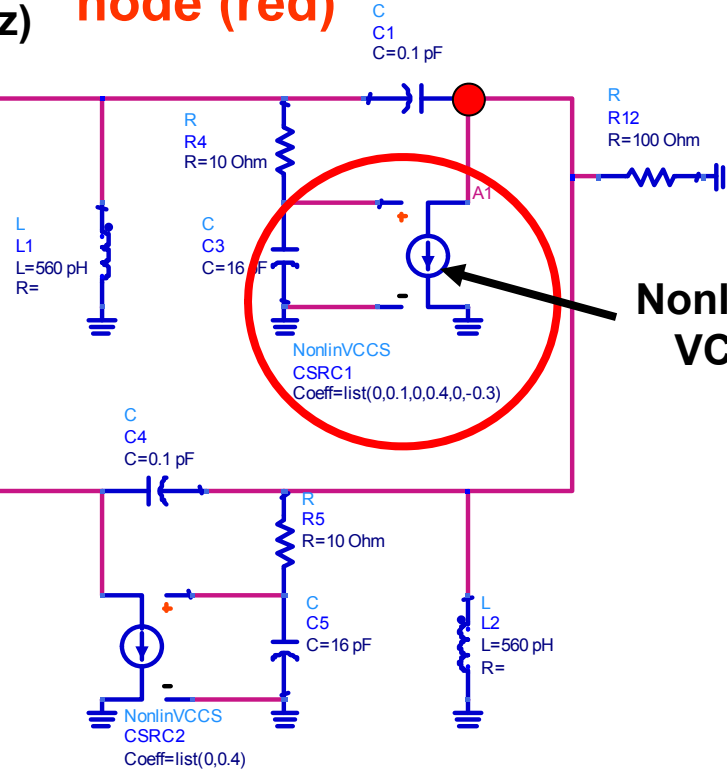
NDF of Nonlinear Ring Oscillator Example (Pin=-50dBm for NDF normalization)

Circuit is known to be stable with zero input drive (from linear NDF analysis)

Large-signal drive tone F1 (1GHz)

Suspect element node (red)

Nonlinear VCCS



```

SWEEP PLAN
SweepPlan
SwpPlan1
Pt=-50
Start=26 Stop=34 Step=0.5 Lin=
UseSweepPlan=
SweepPlan=
Reverse=no
    
```

```

PARAMETER SWEEP
ParamSweep
Sweep6
SweepVar="Pin"
SimInstanceName[1]="HB1"
SimInstanceName[2]=
SimInstanceName[3]=
SimInstanceName[4]=
SimInstanceName[5]=
SimInstanceName[6]=
Start=26
Stop=34
Step=0.5
    
```

```

SWEEP PLAN
SweepPlan
SwpPlan2
Start=0.001 Stop=100 Dec=200 Log=
Start=2.3 Stop=2.5 Step=0.002 Lin=
Start=2.41 Stop=2.43 Step=0.0002 Lin=
Start=3.3 Stop=3.7 Step=0.002 Lin=
Start=3.57 Stop=3.59 Step=0.0002 Lin=
Start=5.2 Stop=5.8 Step=0.002 Lin=
UseSweepPlan=
SweepPlan=
Reverse=no
    
```

```

HARMONIC BALANCE
HarmonicBalance
HB1
MaxOrder=10
Freq[1]=F1 GHz
Freq[2]=F2 GHz
Order[1]=10
Order[2]=1
Oversample[1]=3
Oversample[2]=3
    
```

Max mixing order set to 10

```

P_1Tone
PORT1
Num=1
Z=50 Ohm
P=polar(dbrmtow(Pin),0)
Freq=F1 GHz
    
```

```

VAR
VAR1
Pin=27.5
F1=1.0
F2=1.3
    
```

```

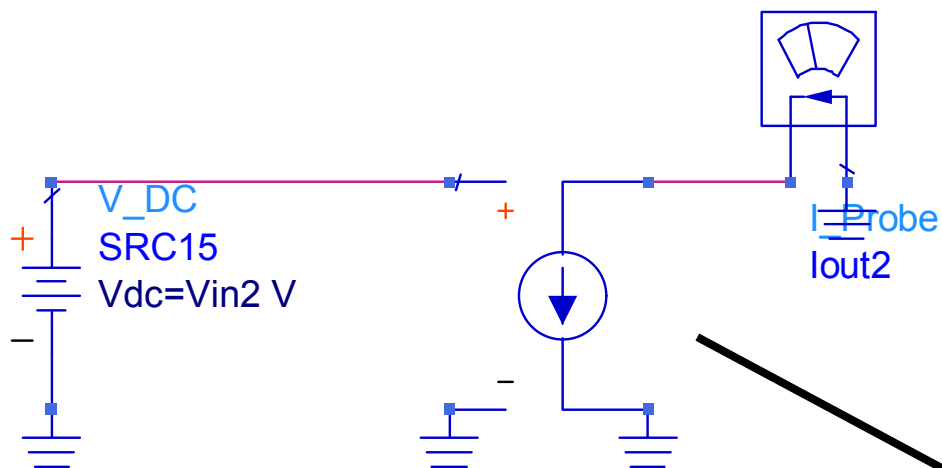
Vfc
Vfc1
V1=vfc(A1,0,{0,1})
    
```

```

L_1Tone
SRC10
I=polar(10,0) uA
Freq=F2 GHz
    
```

Small perturbation current tone F2 (swept freq)

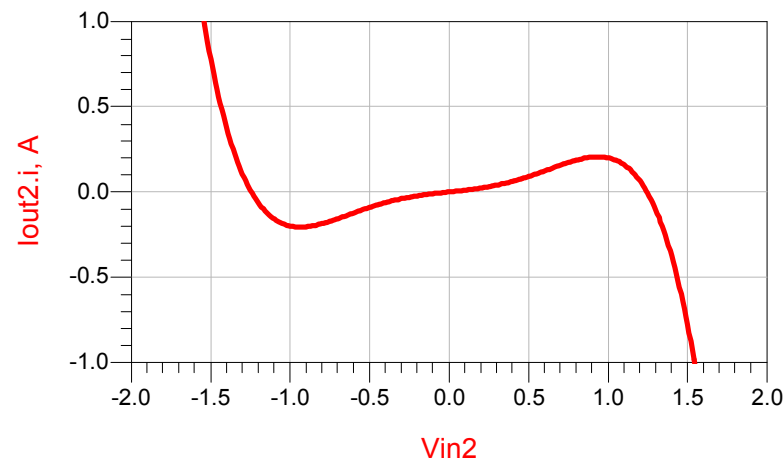
Nonlinear VCCS in Ring Oscillator Example



NonlinVCCS
CSRC3
Coeff=list(0,0.1,0,0.4,0,-0.3)

$$I = 0.1V_{in} + 0.4*V_{in}^3 - 0.3*V_{in}^5$$

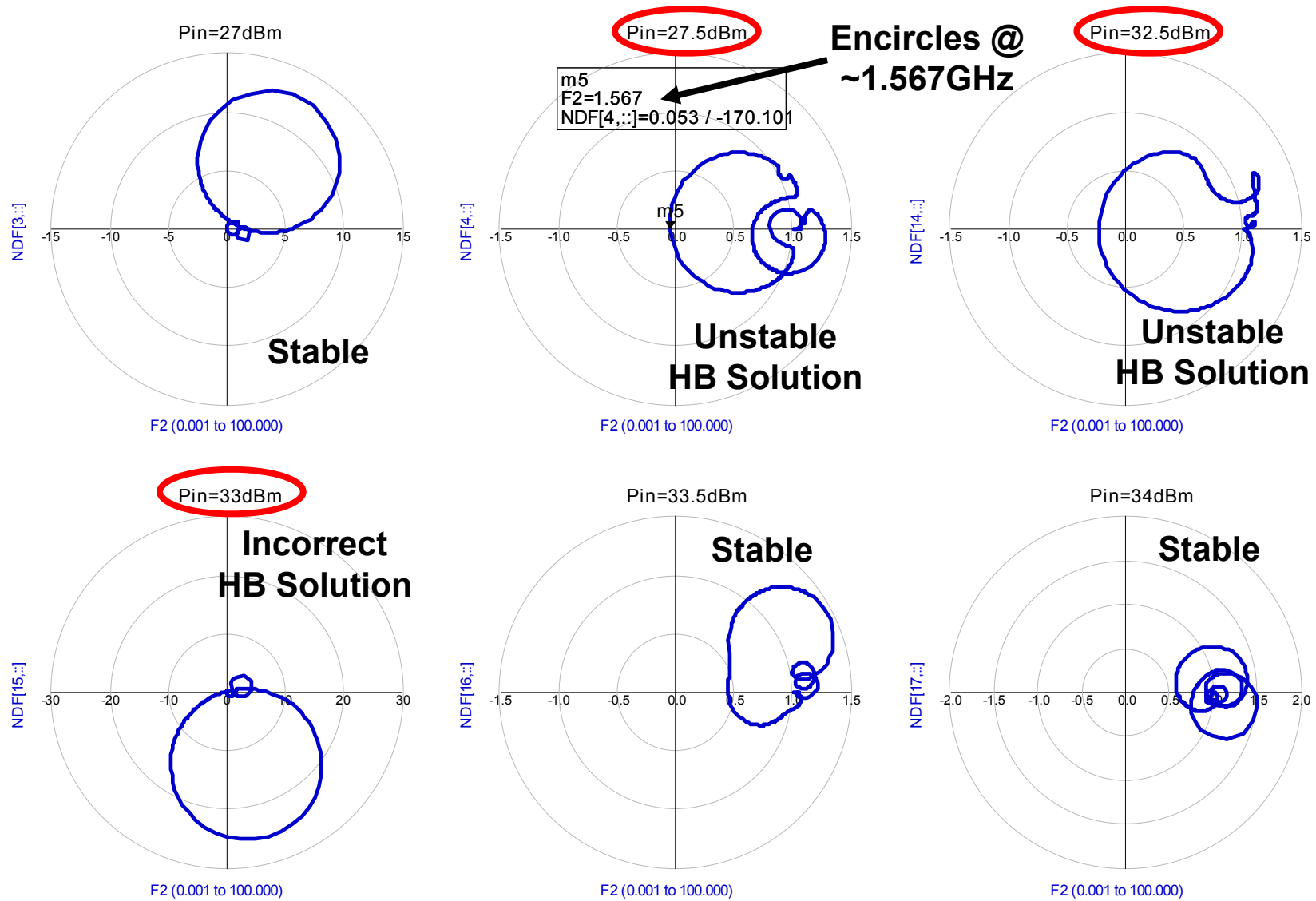
Transconductance (g_m) is a non-linear function of input voltage



Nonlinear Ring Oscillator Example

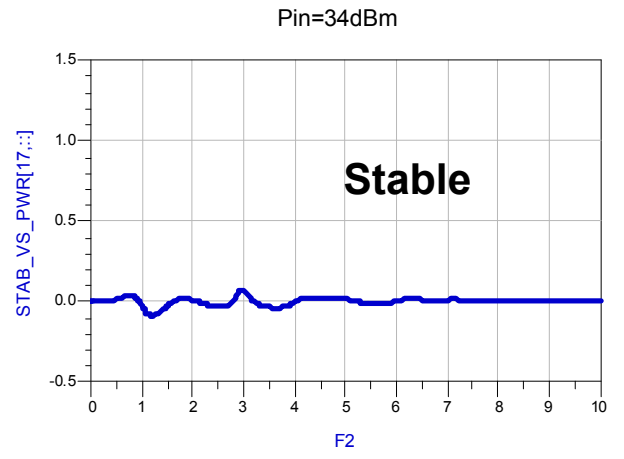
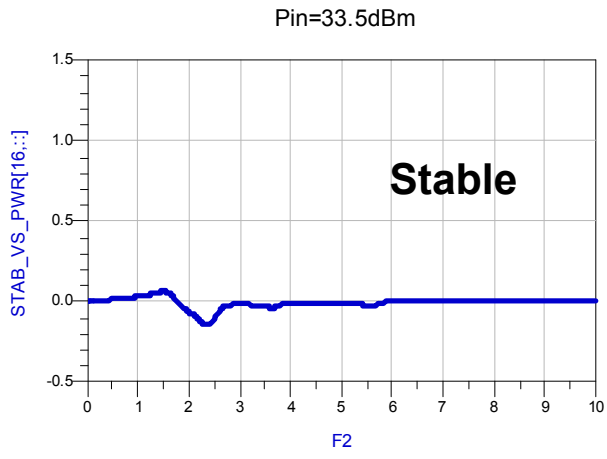
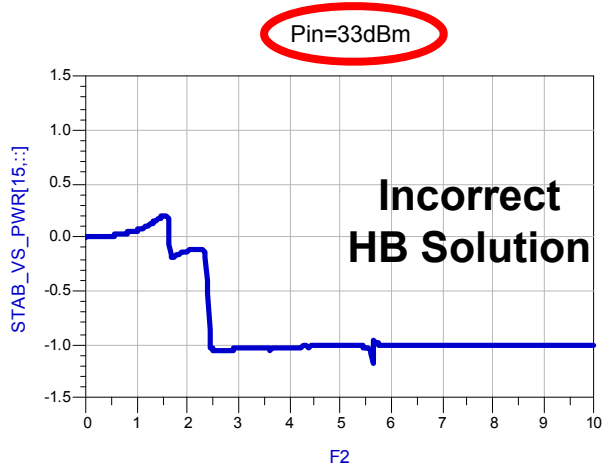
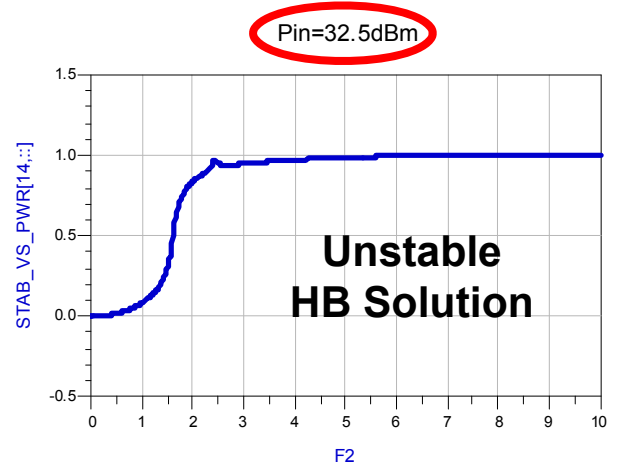
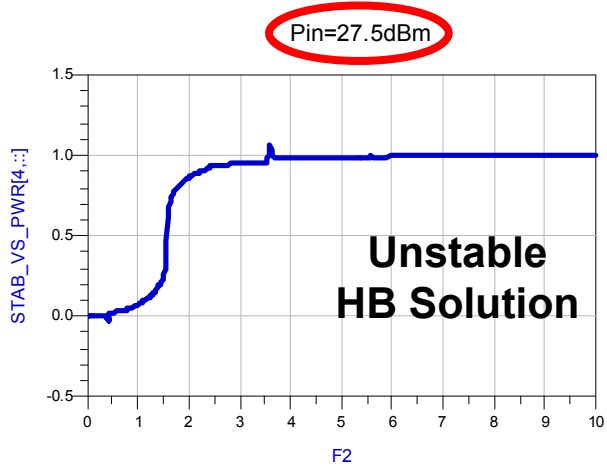
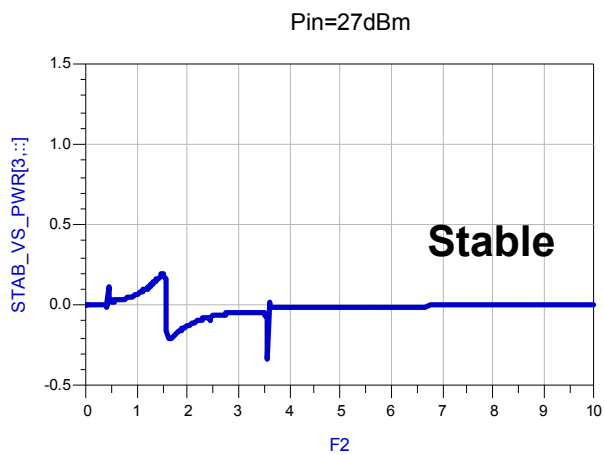
- From the following plots, it is seen that the circuit will oscillate only when driven (at 1GHz) from 27.5 to 33dBm. Outside of this regime, the circuit is stable.
- We verified this by also performing transient simulations of the circuit under various drive powers to confirm this behavior.

NDF Versus Input Power (Drive Freq=1.0GHz) Perturbation Frequency Sweep (Pin=-50dBm for normalization)



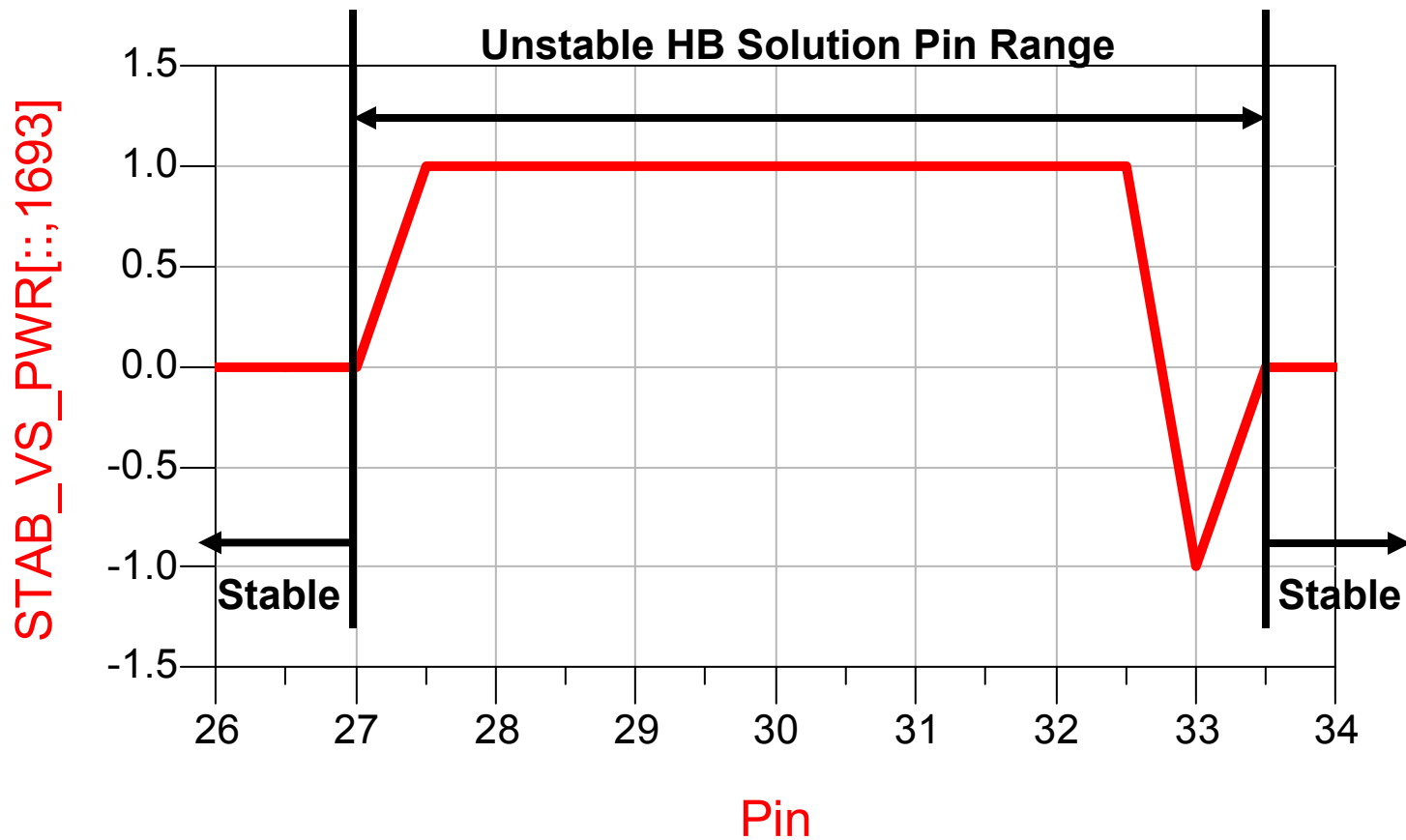
NDF Versus Input Power (Drive Freq=1.0GHz) Perturbation Frequency Sweep (Pin=-50dBm for normalization)

Eqn STAB_VS_PWR=unwrap(phase(NDF))/-360

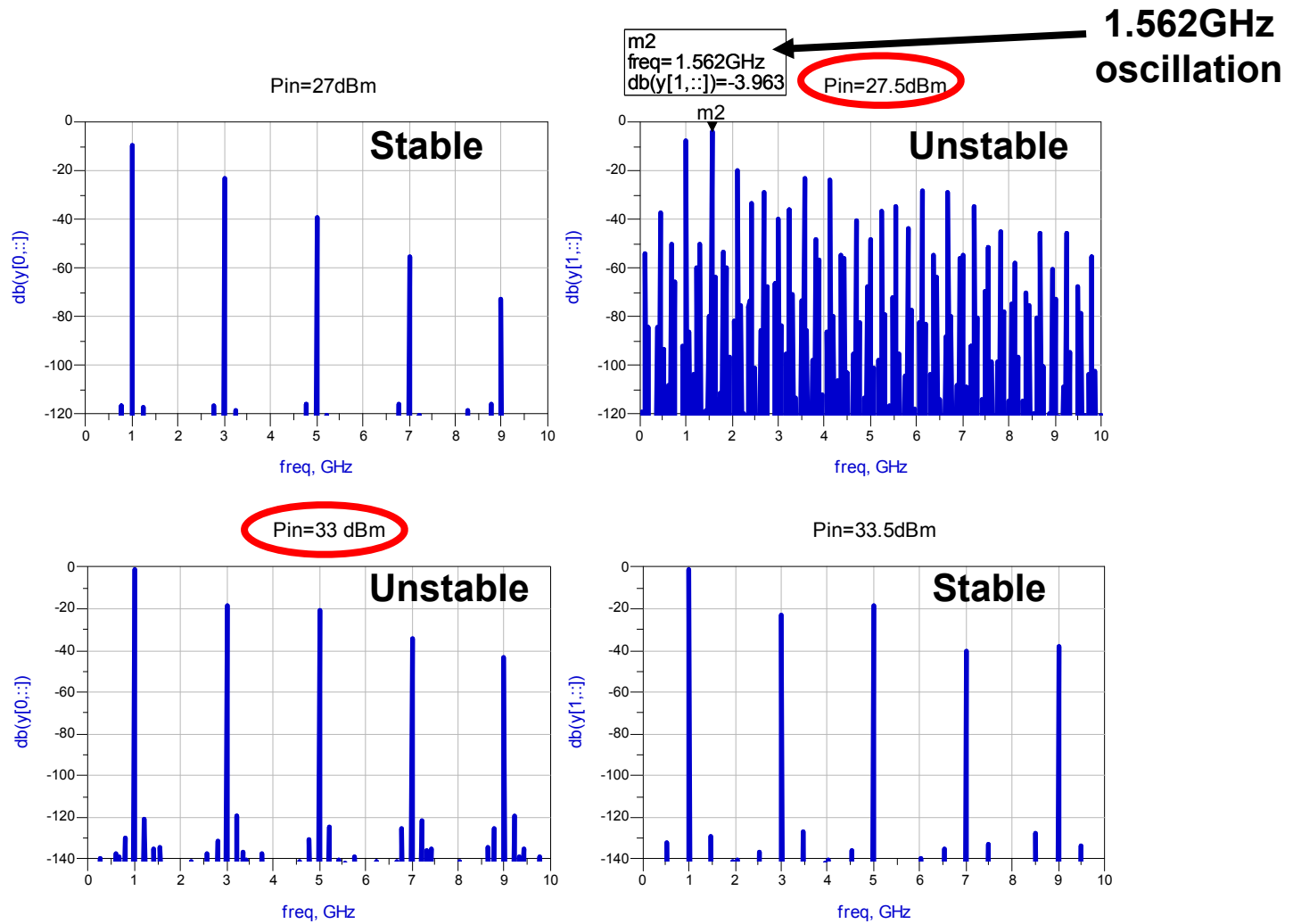


Easier Way to View Stability Versus Pin (plot last frequency point of un-raveled phase $\omega = +\infty$)

Eqn `STAB_VS_PWR=unwrap(phase(NDF))/-360`



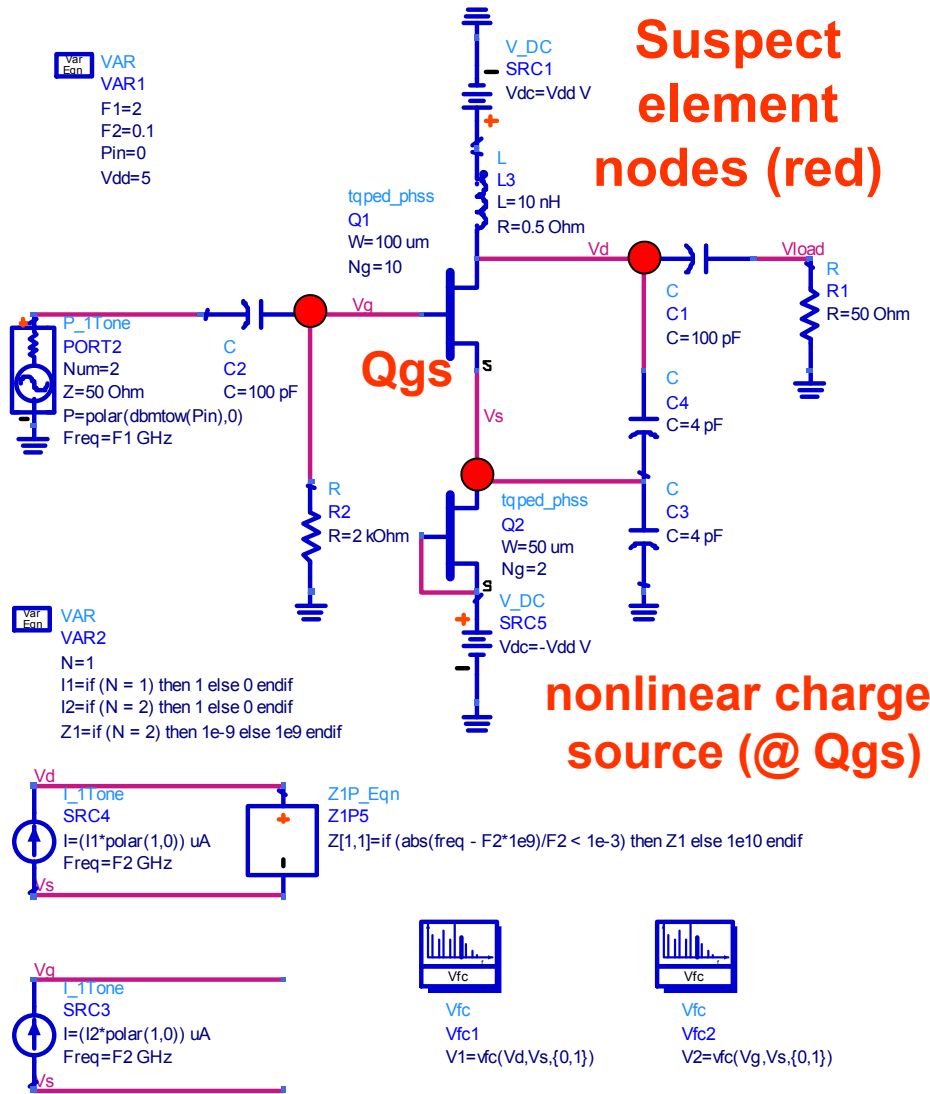
Vout Versus Input Power (Drive Freq=1.0GHz) Transient Analysis (0-20 μ s)



Non-linear NDF Stability Analysis

- Our final example demonstrates that (in the general case) we must include nodes at dependent nonlinear charge sources (as well as dependent nonlinear current and voltage sources) to obtain the nonlinear NDF.
- The example nonlinear network is a parametric $f/2$ frequency divider.
- This circuit uses a single active FET transistor to divide the input frequency over a range of input power and frequency.
- We will use the admittance technique to calculate the nonlinear NDF for $f_{in}=2\text{GHz}$ and power levels from -5dBm to 20dBm .
- This involves four 2-tone HB sweeps (two each across V_{ds} and V_{gs}).
- Again, we will use the same network operating at low power level (known stable at -8dBm) for the NDF normalization.
- At each power level, we will calculate the admittance at V_{ds} first, then short V_{ds} (at the perturbing frequency) to calculate the admittance at V_{gs} . (the order is unimportant, we could do V_{gs} first, as long as V_{gs} nodes are shorted for the subsequent admittance calculations at V_{ds} we will get the same NDF function)

Parametric Frequency Divider (Fin=2GHz)



PARAMETER SWEEP

```
ParamSweep
Sweep1
SweepVar="Pin"
SimInstanceName[1]="Sweep4"
SimInstanceName[2]=
SimInstanceName[3]=
SimInstanceName[4]=
SimInstanceName[5]=
SimInstanceName[6]=
Start=-8
Stop=-3.27
Step=100
```

SWEEP PLAN

```
SweepPlan
SwpPlan2
Pt=-8
Start=-5 Stop=20 Step=1 Lin=
UseSweepPlan=
SweepPlan=
Reverse=yes
```

SWEEP PLAN

```
SweepPlan
SwpPlan1
Start=2.9001 Stop=3.1001 Step=0.001 Lin=
Start=4.9001 Stop=5.1001 Step=0.001 Lin=
Start=6.9601 Stop=7.0401 Step=0.0002 Lin=
Start=0.001 Stop=99.99 Dec=200 Log=
UseSweepPlan=
SweepPlan=
Reverse=no
```

HARMONIC BALANCE

```
HarmonicBalance
HB1
MaxOrder=10
Freq[1]=F1 GHz
Freq[2]=F2 GHz
Order[1]=10
Order[2]=1
Oversample[1]=3
Oversample[2]=3
```

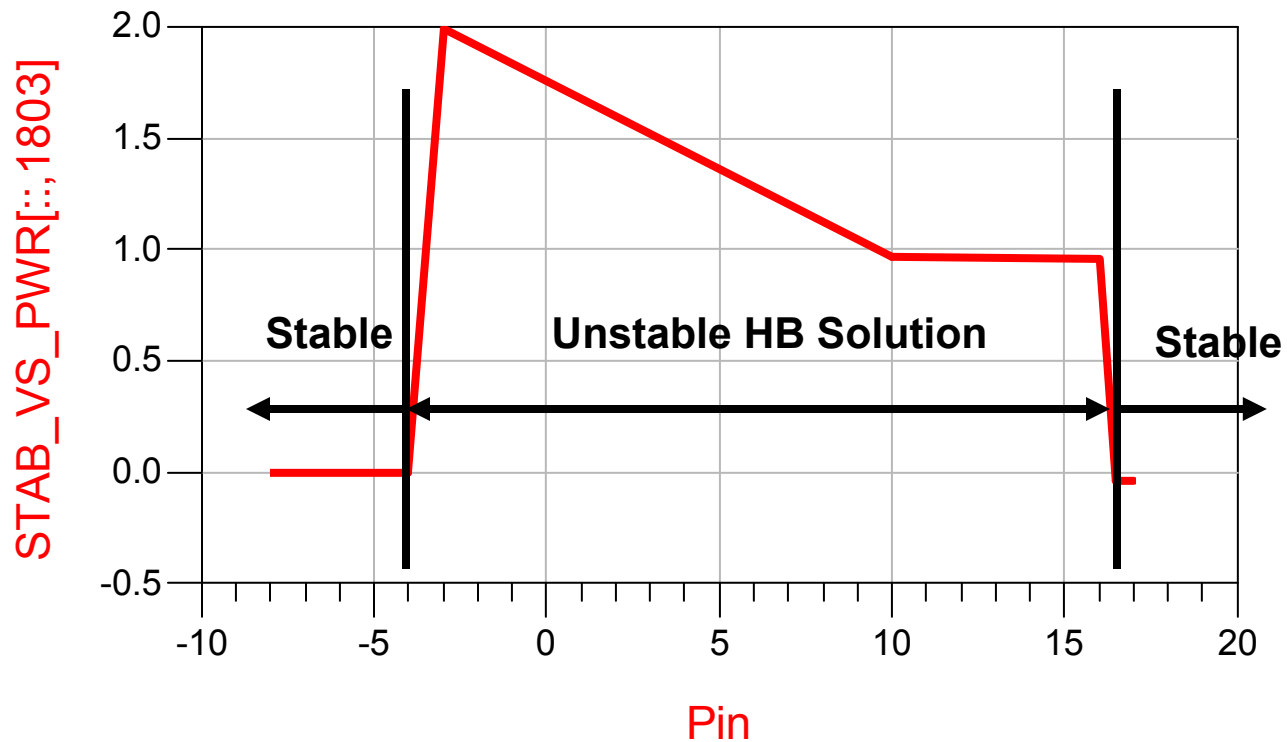
Max Mixing
Order 10

PARAMETER SWEEP

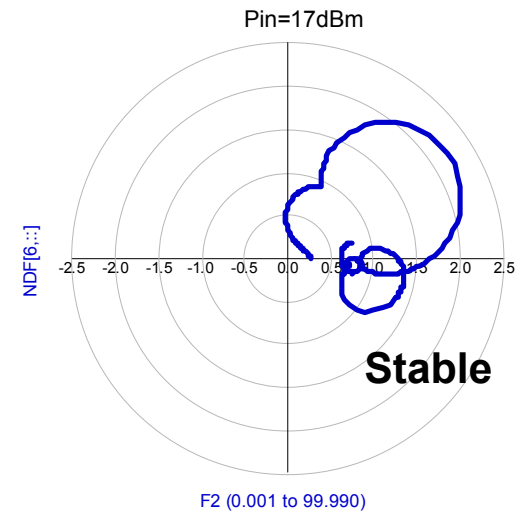
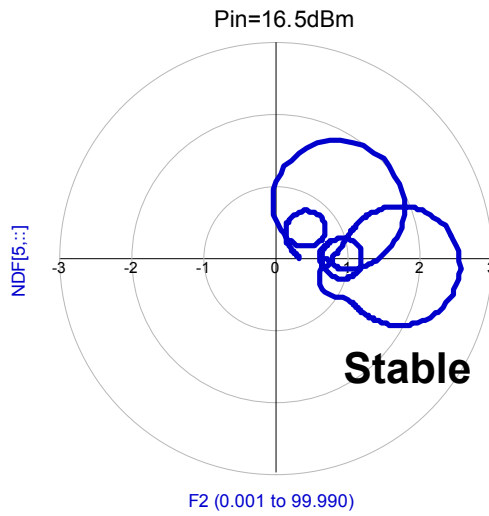
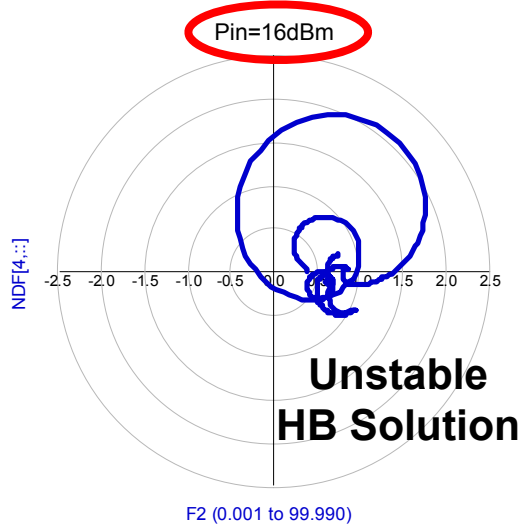
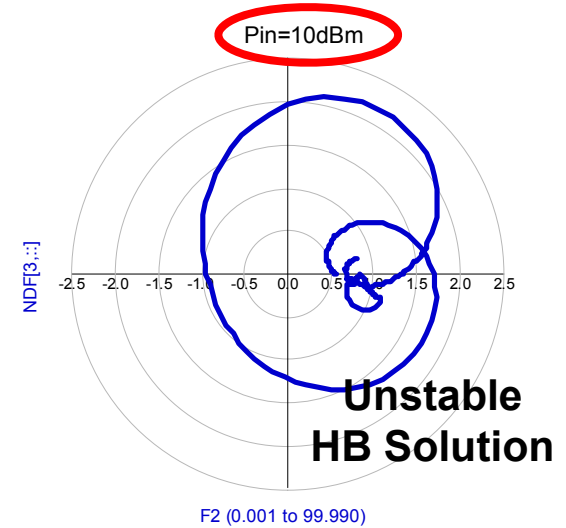
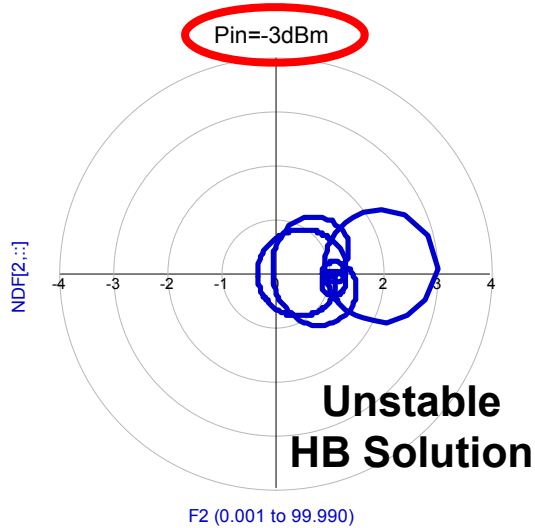
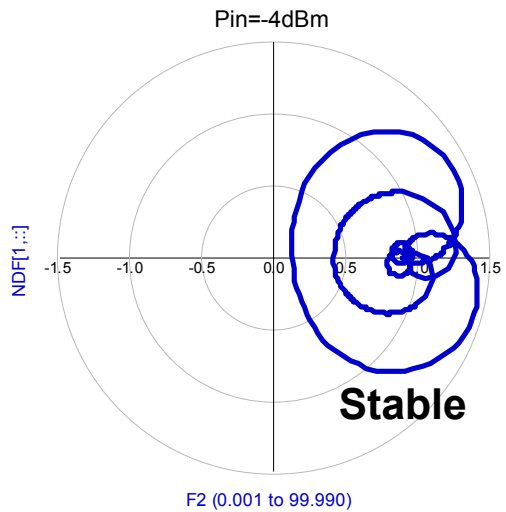
```
ParamSweep
Sweep4
SweepVar="N"
SimInstanceName[1]="HB1"
SimInstanceName[2]=
SimInstanceName[3]=
SimInstanceName[4]=
SimInstanceName[5]=
SimInstanceName[6]=
Start=1
Stop=2
Step=1
```

NDF Stability Versus Pin Encirclement Plot (plot last frequency point of un-raveled phase $\omega = +\infty$)

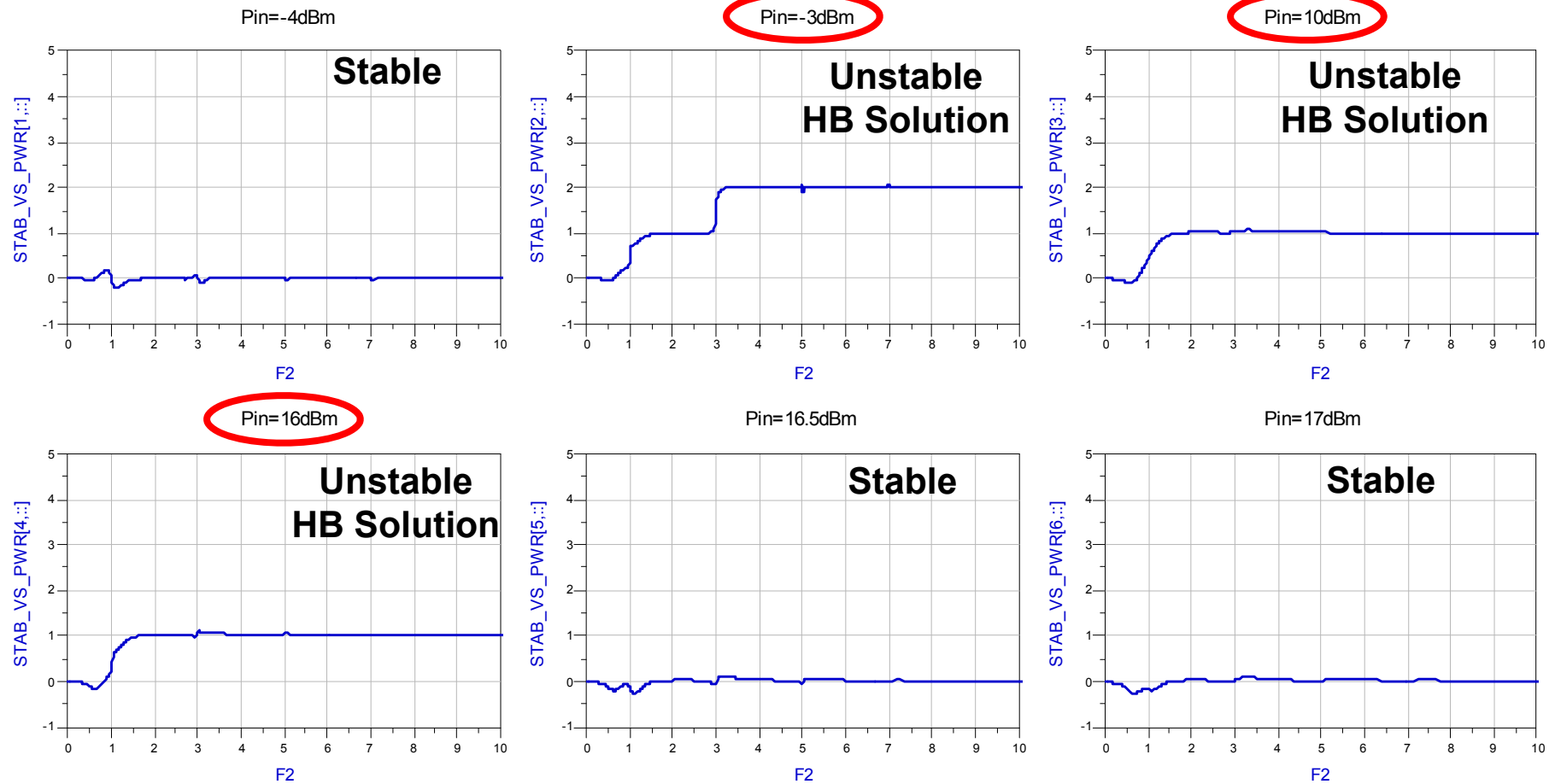
Eqn `STAB_VS_PWR=unwrap(phase(NDF))/-360`



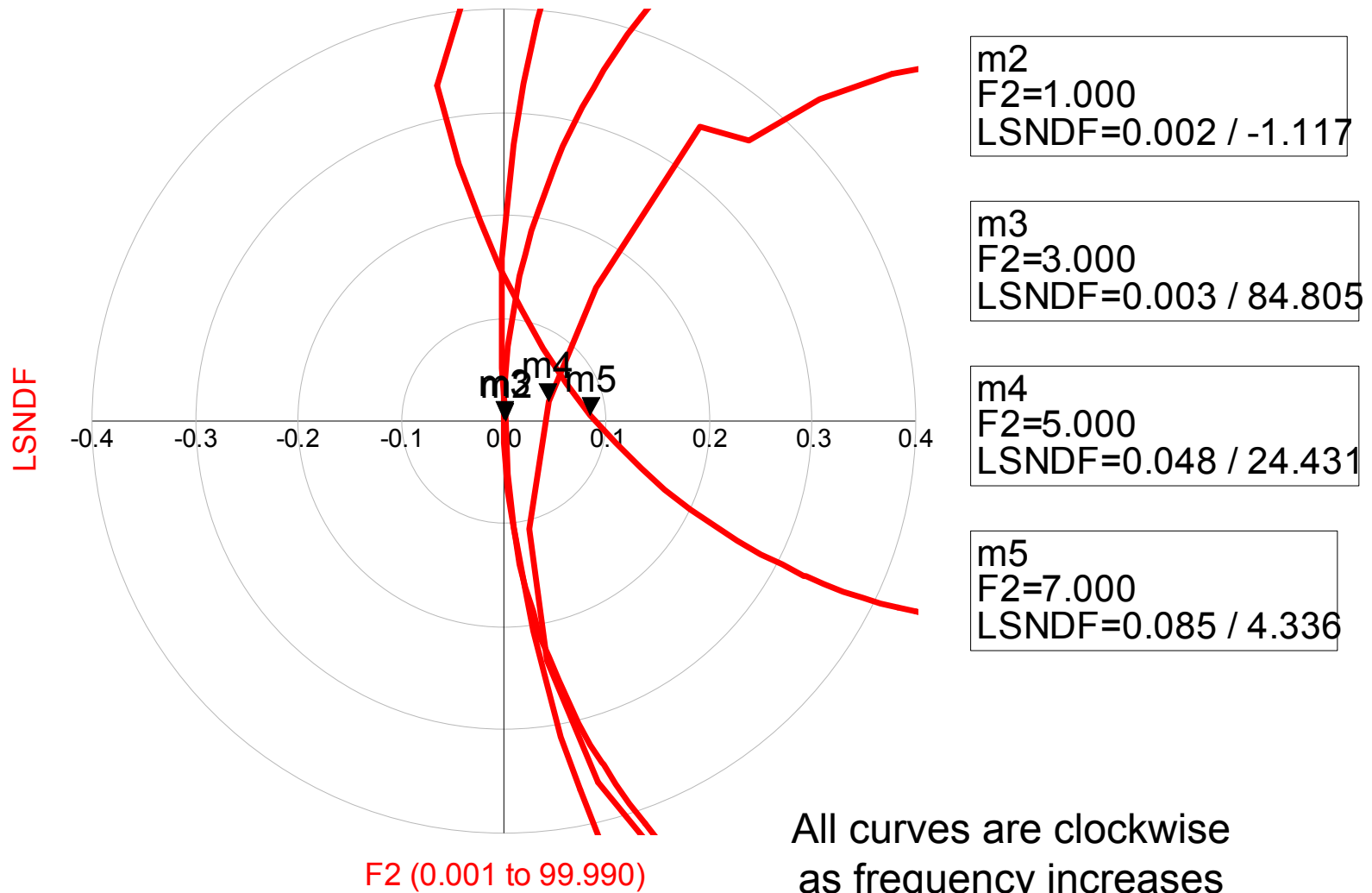
NDF Versus Input Power ($F_{in}=2\text{GHz}$) Perturbation Frequency Sweep ($P_{in}=-8\text{dBm}$ for normalization)



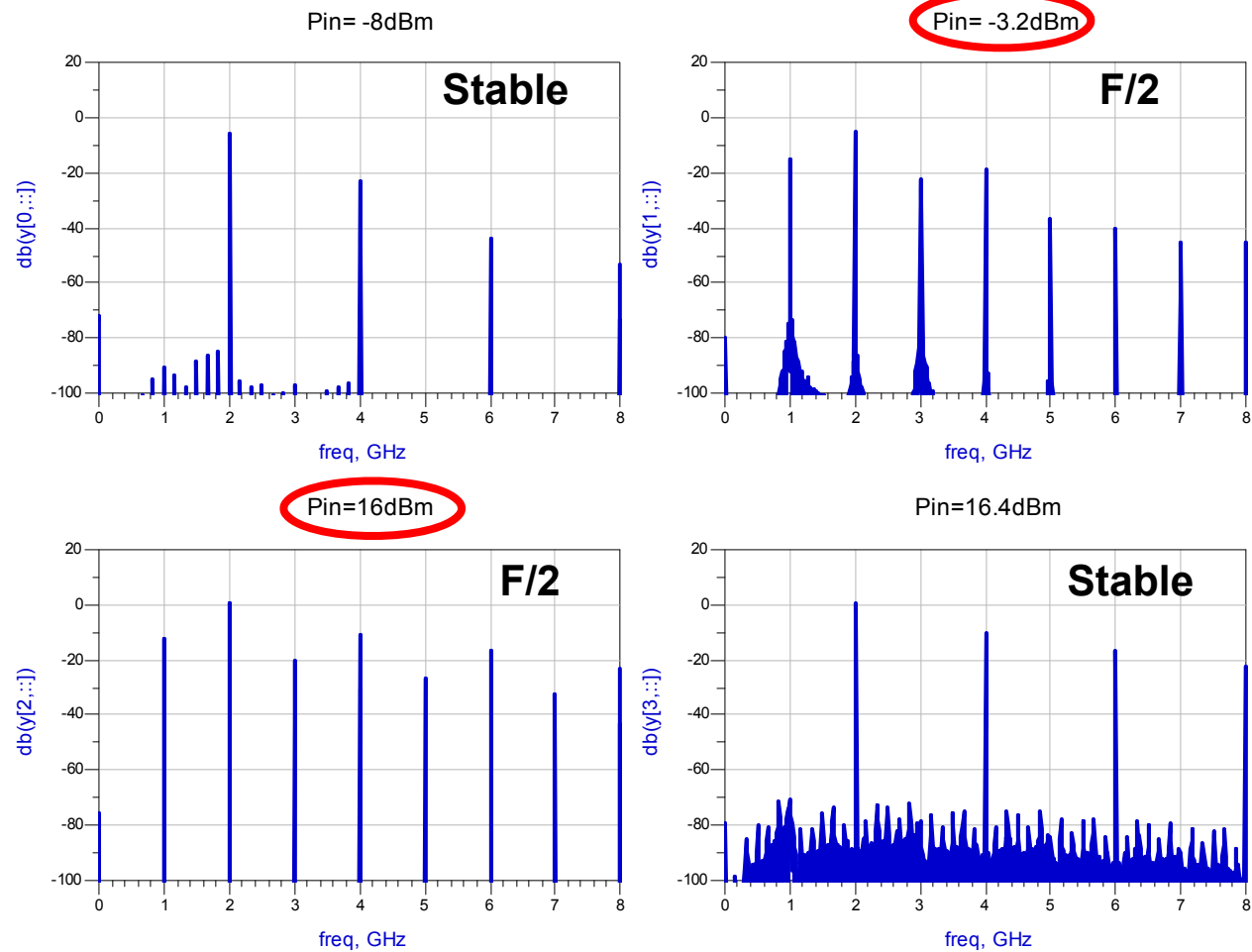
NDF Versus Input Power ($F_{in}=2\text{GHz}$) Perturbation Frequency Sweep ($P_{in}=-8\text{dBm}$ for normalization)



Fin=2GHz Pin=-3.295dBm (frequency division confirmed)
(1,3,5,7GHz go through (0,0) at the same input power level)



Vload Versus Input Power (Fin=2GHz) Transient Analysis (0-10 μ s)



Summary

- A “fairly simple” NDF technique for detecting oscillations in both linear and non-linear networks has been shown.
- Examples using ADS (Advanced Design System) and AWR were shown, however, any linear/non-linear simulator will work just as well.
- This technique is pretty powerful and can detect all types of oscillations including traditional linear circuit oscillations as well as non-linear, power dependent parametric (as well as sub-harmonic) oscillations.
- Keep in mind however, that this technique is only as good as your circuit models. In other words, it won't accurately predict oscillatory behavior if your circuit model isn't accurate or incomplete (see the last reference paper [9] on the next page for a real world example...).
- The technique is however, mathematically rigorous.
- I hope this helps resolve (and prevents) any future non-desired circuit behavior. Good luck and have fun! (you can blame me later...)

References

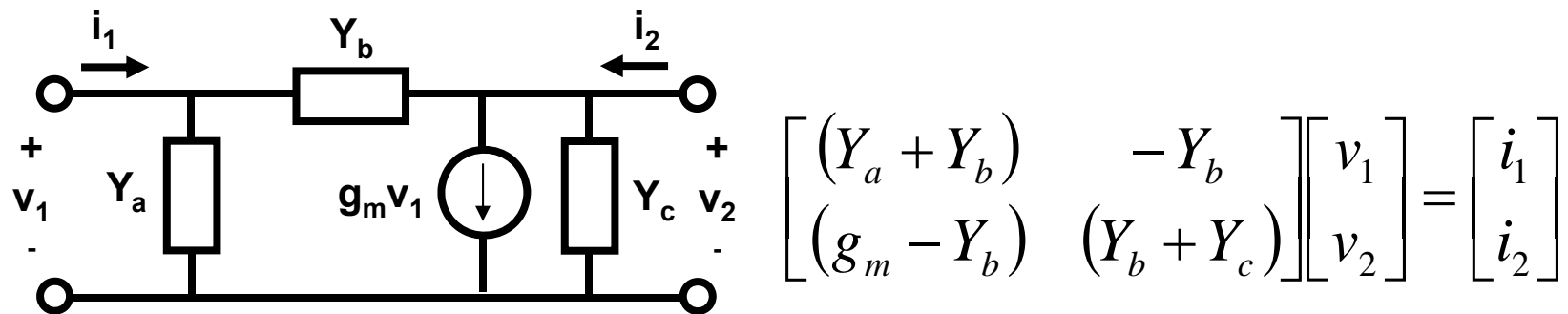
- [1] E. Routh, "Dynamics of a System of Rigid Bodies", 3rd Ed., Macmillan, London, 1877
- [2] H. Nyquist, "Regeneration Theory", Bell System Technical Journal, Vol. 11, pp. 126-147, Jan. 1932
- [3] H. Bode, "Network Analysis and Feedback Amplifier Design", D. Van Nostrand Co. Inc., New York, 1945
- [4] D.J.H. Maclean, "Broadband Feedback Amplifiers", Research Studies Press, 1982
- [5] A. Platzker, W. Struble, and K. Hetzler, "Instabilities Diagnosis and the Role of K in Microwave Circuits", IEEE MTT-S Digest, vol. 3, pp. 1185-1188, Jun. 1993
- [6] W. Struble and A. Platzker, "A Rigorous Yet Simple Method For Determining Stability of Linear N-port Networks", 15th Annual GaAs IC Symposium Digest, pp. 251-254, Oct. 1993
- [7] A. Platzker and W. Struble, "Rigorous Determination of The Stability of Linear N-node Circuits From Network Determinants and The Appropriate Role of The Stability Factor K of Their Reduced Two-Ports", 3rd International Workshop on Integrated Nonlinear Microwave and Millimeterwave Circuits, pp. 93-107, Oct. 1994
- [8] J. Jugo, J. Portilla, A. Anakabe, A. Suarez and J.M. Collantes, "Closed-Loop Stability Analysis of Microwave Amplifiers", Electronics Letters, vol.37 No. 4, pp. 226-228, Feb. 2001
- [9] C. Barquinero, A. Suarez, A. Herrera and J.L. Garcia, "Complete Stability Analysis of Multifunction MMIC Circuits", IEEE Tran. Microwave Theory and Techniques, vol.55 No. 10, pp. 2024-2033, Oct. 2007

Dead (passive) “black box” Transistor Models

Wayne Struble
and
Aryeh Platzker

Dead “black box” Transistor Models

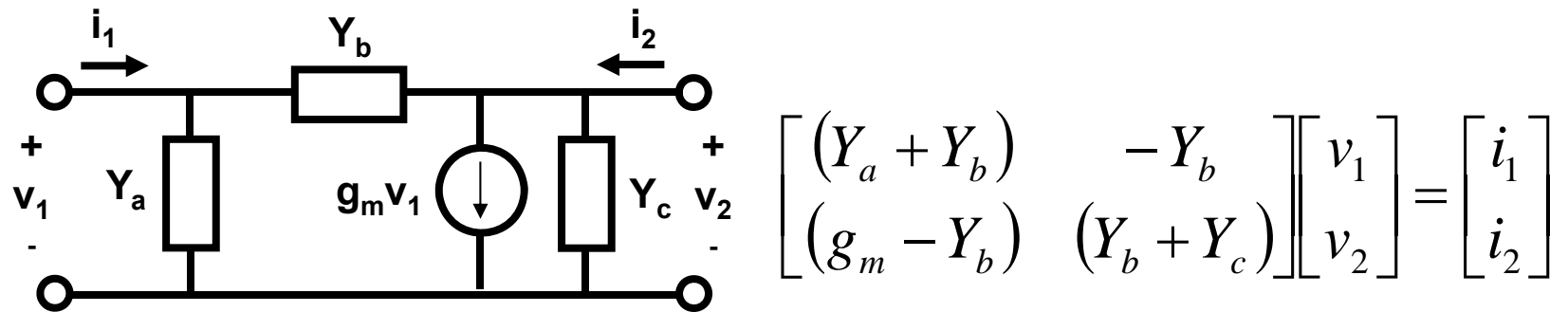
- How do we make a “black box” transistor in a linear network passive (no RHP zeroes).
- Consider the simplified network representation of an active transistor **(at any single given frequency)** (D.J.H. Maclean [1]). This is valid for any transistor type **(FET or Bipolar)**. (can also be a [S]-parameter file)



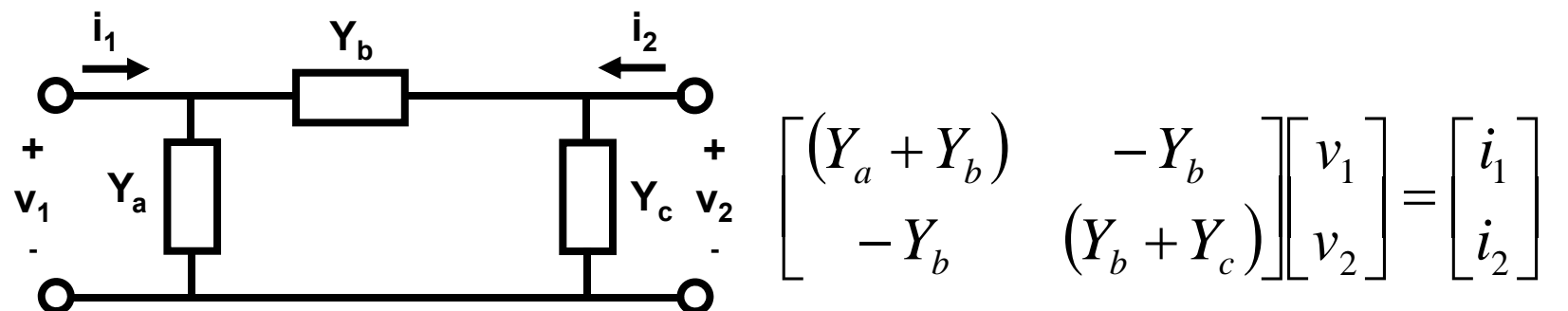
$$(Y_a + Y_b)v_1 - Y_b v_2 = i_1$$

$$(g_m - Y_b)v_1 + (Y_b + Y_c)v_2 = i_2$$

Dead “black box” Transistor Models

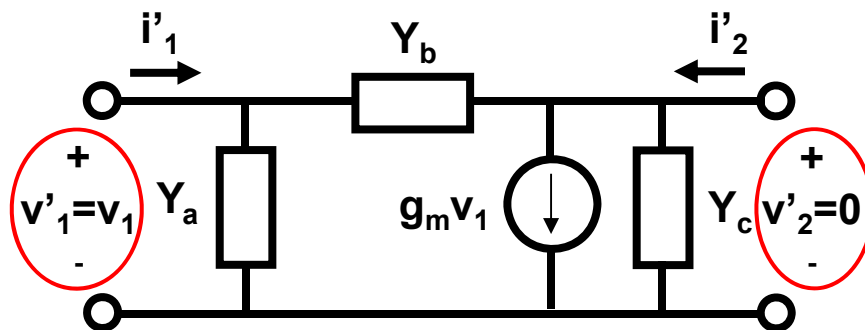


- We want to set $g_m = 0$ at all frequencies (other than DC) to make the transistor passive.



Dead “black box” Transistor Models

- We need to subtract $g_m v_1$ from i_2 in the simplified model to achieve this. This is done using two additional (identical) transistors that use the following AC terminal conditions (DC are the same as original):
- First, in identical transistor 1, set the **input voltage** equal to v_1 (sampled from the original transistor terminals), short v_2 and find the short circuit current i_2 .

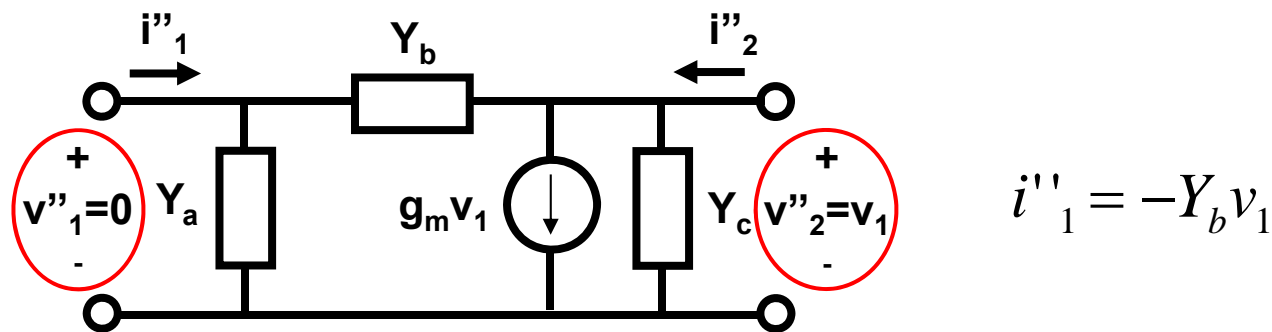


$$i'_2 = (g_m - Y_b)v_1$$

- The short circuit current is $(g_m - Y_b)v_1$.
- Next we need to get $Y_b v_1$.

Dead “black box” Transistor Models

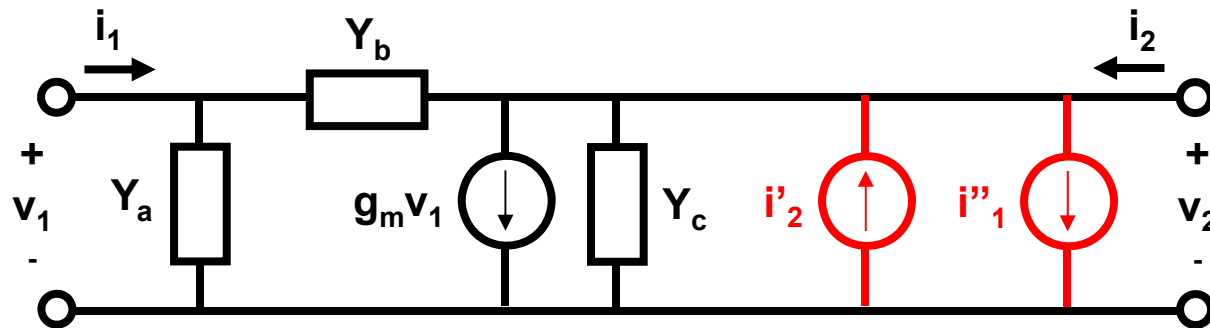
- Next, in identical transistor 2, set the **output voltage** equal to v_1 (sampled from the original transistor terminals) and find the short circuit input current i_1 .



- The short circuit current is $-Y_b v_1$.
- Next we simply subtract i_2' and add i_1'' back to original transistor model as shown on the next slide.

Dead “black box” Transistor Models

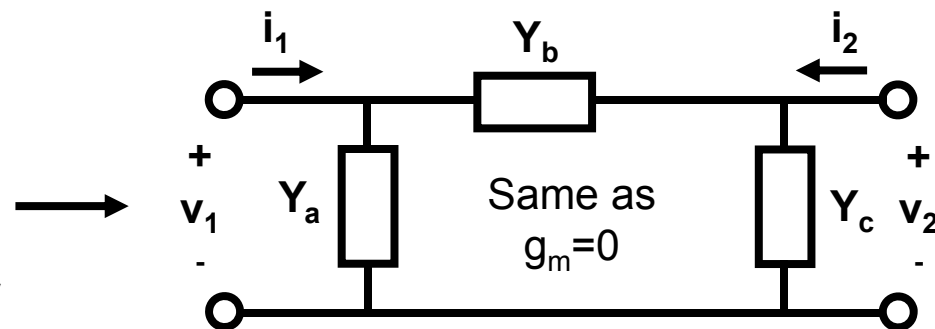
- Subtract i'_2 and add i''_1 to the original model like so...



$$\begin{aligned} (Y_a + Y_b)v_1 - Y_b v_2 &= i_1 & i'_2 &= (g_m - Y_b)v_1 \\ (g_m - Y_b)v_1 + (Y_b + Y_c)v_2 - i'_2 + i''_1 &= i_2 & i''_1 &= -Y_b v_1 \end{aligned}$$

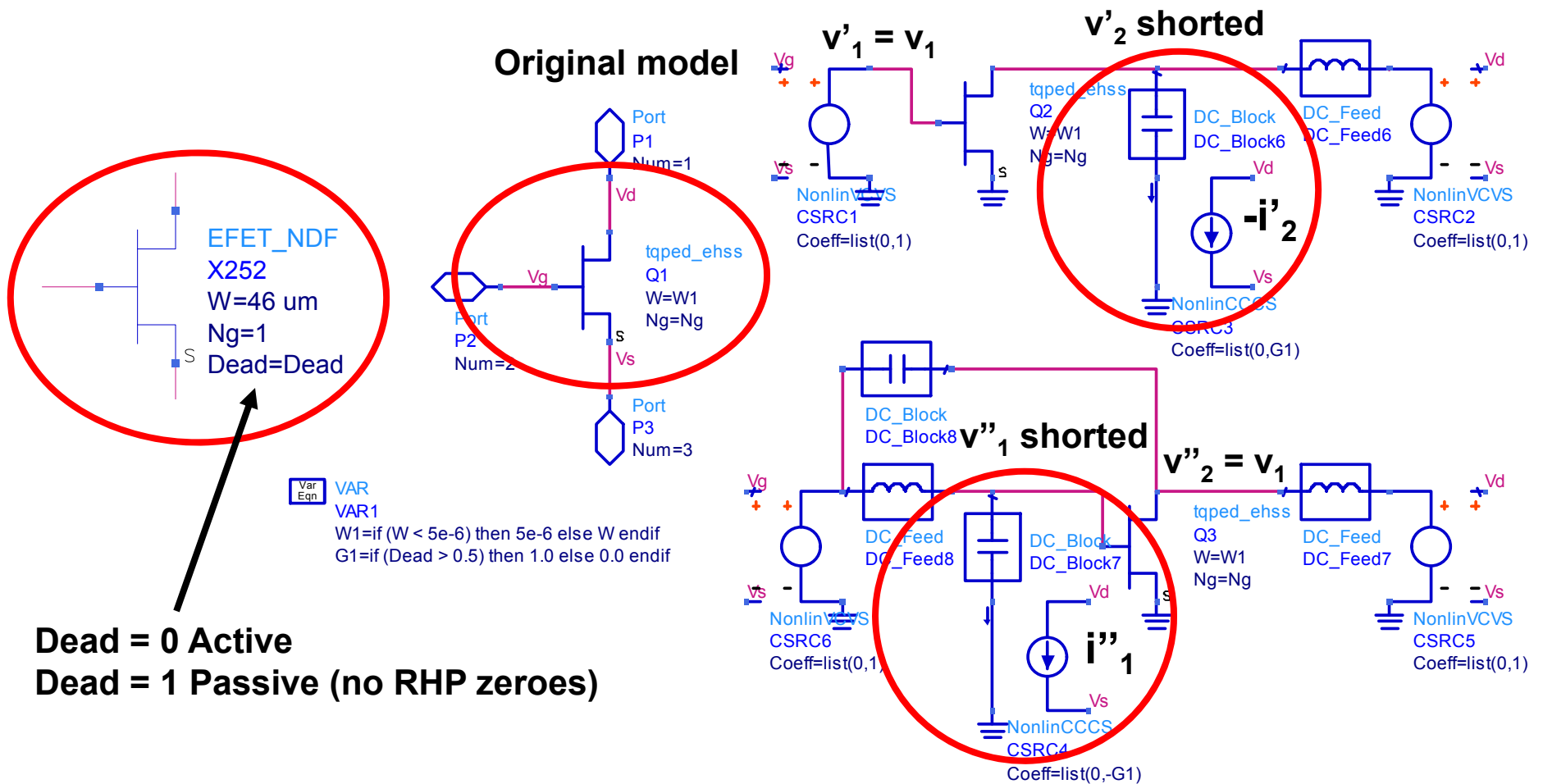
This results in ↓

$$\begin{aligned} (Y_a + Y_b)v_1 - Y_b v_2 &= i_1 \\ -Y_b v_1 + (Y_b + Y_c)v_2 &= i_2 \end{aligned}$$



ADS Implementation of “dead” Transistor Models (uses 3 identical transistor models)

- Model gives same DC currents as active model, but AC $g_m = 0$ (no gain at AC frequencies). Same can be done for BJTs, [S]-parameter files.



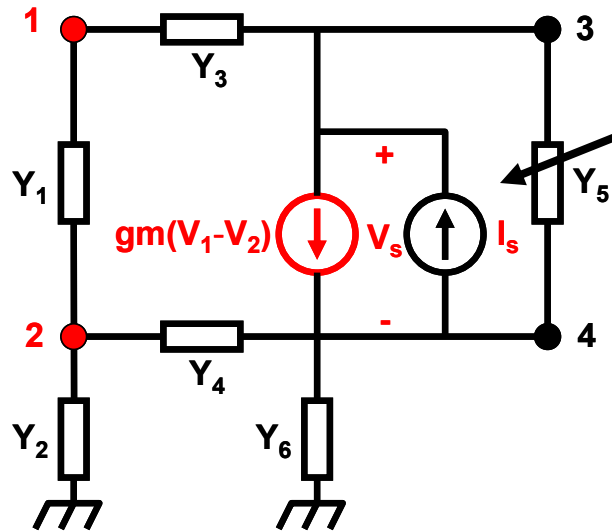
References

- [1] D.J.H. Maclean, "Broadband Feedback Amplifiers", Research Studies Press, 1982

NDF Stability Analysis of Linear Networks from Network Admittances

**Wayne Struble
and
Aryeh Platzker**

Relationship Between Network Determinants and Admittance



Add current source I_s
in parallel with dependent
Source and measure V_s

$$\text{Admittance} = I_s / V_s$$

$$[Y][V] = [I]$$

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & 0 & -Y_4 \\ gm - Y_3 & -gm & Y_3 + Y_5 & -Y_5 \\ -gm & gm - Y_4 & -Y_5 & Y_4 + Y_5 + Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_s \\ -I_s \end{bmatrix}$$

Relationship Between Network Determinants and Admittance

- First redefine the node voltages in terms of V_s where $V_4 = V_3 - V_s$
- So, column 3 in the matrix becomes the sum of columns 3 and 4, and column 4 changes sign as shown below.

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 & Y_4 \\ gm - Y_3 & -gm & Y_3 & Y_5 \\ -gm & gm - Y_4 & Y_4 + Y_6 & -Y_4 - Y_5 - Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_s \\ -I_s \end{bmatrix}$$

Relationship Between Network Determinants and Admittance

- Next, we replace row 3 in the matrix with the sum of rows 3 and 4 to eliminate I_s on the right side.

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 & Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 & -Y_4 - Y_6 \\ -gm & gm - Y_4 & Y_4 + Y_6 & -Y_4 - Y_5 - Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_s \end{bmatrix}$$

- And multiplying row 4 by -1,

$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 & 0 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 & Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 & -Y_4 - Y_6 \\ gm & -gm + Y_4 & -Y_4 - Y_6 & Y_4 + Y_5 + Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_s \end{bmatrix}$$

Relationship Between Network Determinants and Admittance

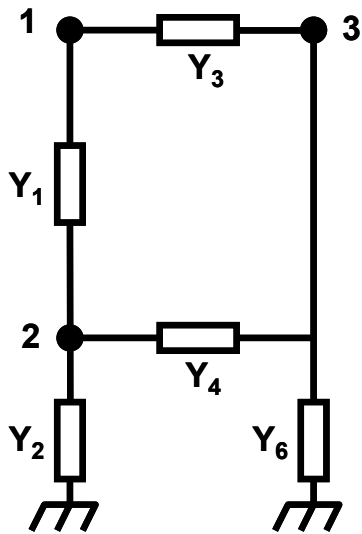
- Now using Cramer's rule.

$$V_s = \frac{I_s \begin{vmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 \end{vmatrix}}{|Y|}$$

- Next, we recognize that the determinant in the numerator is equal to the determinant of the original network with nodes 3 and 4 shorted (i.e. dependent source is shorted)...

Relationship Between Network Determinants and Admittance

- Original network with nodes 3 & 4 shorted



$$\begin{bmatrix} Y_1 + Y_3 & -Y_1 & -Y_3 \\ -Y_1 & Y_1 + Y_2 + Y_4 & -Y_4 \\ -Y_3 & -Y_4 & Y_3 + Y_4 + Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

- Therefore the admittance,

$$A = \frac{I_s}{V_s} = \frac{|Y|}{|Y|_{(V_3 \& V_4 \text{ shorted})}}$$

- Note that this shorted network is passive (no RHP zeroes), but is not of the same order as the original network.

Relationship Between Network Determinants and Admittance

- Next, we calculate admittance (at the same nodes) from a known stable operating point (gm=0 for example).

$$A_0 = \frac{I_s}{V_s} = \frac{|Y_{stable}|}{|Y_{stable}|_{(V_3 \& V_4 \text{shorted})}} \quad A = \frac{I_s}{V_s} = \frac{|Y|}{|Y|_{(V_3 \& V_4 \text{shorted})}}$$

- Since, $|Y_{stable}|_{(V_3 \& V_4 \text{shorted})} = |Y|_{(V_3 \& V_4 \text{shorted})}$
- Because the dependent source has been shorted.

$$NDF = \frac{A}{A_0} = \frac{|Y|}{|Y_{stable}|}$$